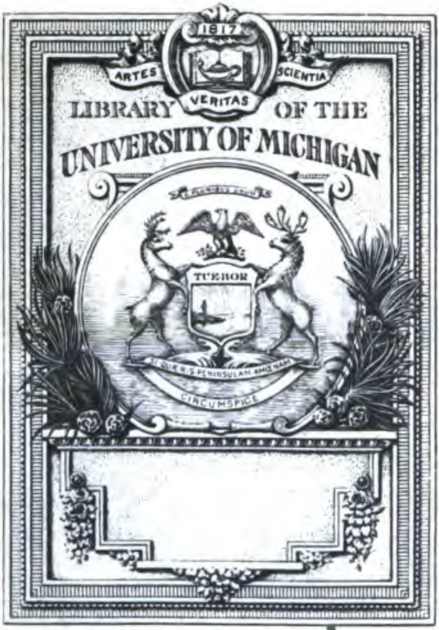
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COBB'S

EXPLANATORY

ARITHMETICK,

NUMBER TWO:

CONTAINING THE

COMPOUND RULES, AND ALL THAT IS NECESSARY OF EVERY OTHER RULE IN ARITHMETICK FOR PRACTICAL PURPOSES AND THE TRANSACTIONS OF BUSINESS:

IN WHICH

THE PRINCIPLES OF EACH RULE ARE FULLY AND FAMILIARLY EXPRESSED; AND THE REASONS FOR EVERY OPERATION ON THE SLATE MINUTELY AND CLEARLY EXPLAINED; AND ADAPTED TO THE UNDERSTANDING AND USE OF LARGER CHILDREN, IN SCHOOLS, AND ACADEMIES.

TO WHICH IS ANNEXED

A PRACTICAL SYSTEM OF BOOK-KEEPING.

BY LYMAN COBB,

AUTHOR OF THE SPELLING-BOOK, SCHOOL DICTIONARY, EXPOSITOR, JUVENILE READERS, SEQUEL, NORTH AMERICAN READER, AND CIPHERING-BOOK, NOS. 1 AND 2.

PHILADELPHIA,
DESILVER, THOMAS, & CO.
CHAMBERSBURGH, PA.
HICKOK & BLOOD.
1836.

PREFACE.

THE Author of the following treatise is fully sensible, that in offering a new Work to the publick there is a degree of assuming confidence necessarily implied, more particularly if others have written on the same subject. He is also aware, that every community experiences the important advantages arising from a useful education of the individuals who compose it; and that, therefore, he who offers any thing which has a tendency to promote this great object, or facilitate the means of acquiring it, has some claim to attention.

The Author has, from his own observation and experience, long since been convinced that, among the many introductory books to the useful science of Arithmetick, no one, or at least none with which he is acquainted, is sufficiently adapted to the capacities of children, and to the occasions of common life. Some are too abstruse for young beginners, while others are deficient in such examples as point out the application of the several rules to transactions of real business.

In presenting a new System of Arithmetick to the publick, some account of its plan and execution will, of course, be expected.

The Author of this work has endeavoured to furnish a clear and familiar description of the rules of Arithmetick, and to introduce the learner to this pleasing and very valuable art, by gradually unfolding to him the modes of practice, and the principles on which the several rules proceed, in plain and intelligible language; and in order to render the rules still more clear and familiar to the learner, and also to encourage him, the

first question in each rule is worked at full length, so that he is led forward in a gradual manner, both by precept and example; and is, it is conceived, more fully convinced of the truth of the rules, and the propriety of each operation.

In this work the Table belonging to each rule is first given, that the scholar may learn the nature of the rule before he is questioned respecting it, or has sums given him for exercise. Examples for *mental* exercise are then given to awaken his reasoning powers, and thus prepare him to engage in each rule theoretically. The rule is then given in questions and answers, as this form, the Author believes, is better calculated to illustrate the rule, and to impress it on the mind of the pupil. Examples are then given for *theoretical* exercise on a slate, and one or more of these examples is worked out, and every operation fully and minutely explained. The remaining examples, not worked out, are for the exercise of the learner, until he shall become thoroughly acquainted with the theory of each rule. Immediately after these, are examples for *practical* exercise, consisting of a variety of miscellaneous questions, in which will be found much useful information.

Most children, it is believed, experience some disgust in passing through the fundamental, or first five rules of Arithmetick, occasioned, no doubt, by the fewness of examples, and by the want of interest in those that are given. An Arithmetick should not consist, as is most generally the case, merely of an assemblage of rules and examples without EXPLANATION; so that the learner, after having committed them to memory, and learned to perform Arithmetical calculations mechanically, will leave the study totally ignorant of the principles upon which the rules are founded. Pupils are always desirous of knowing the reasons *why* any Arithmetical operation is

performed; and if the nature and principles of the subject are clearly explained, and the rule rendered intelligible by the Author, the scholar may be able to acquire a knowledge of it without much aid from the teacher. The study would then be pleasant, and he would pursue it with delight and profit.

The rules which the scholar should commit to memory are in the largest type used in the work. The examples, explanations, and exercises, are in a type of a smaller size; and the notes intended for the teacher are in the smallest type. The EXPLANATIONS should be thoroughly and carefully read by the scholar.

The learner should be questioned as often as once in each day respecting the principles upon which the rules are founded; and the teacher should not permit him to commence a new sum, or engage in a new rule, until he is fully and thoroughly acquainted with the principles of the rule in which he has been working. Young scholars are generally anxious to make rapid progress. This propensity, however laudable, should not be indulged at the expense of a partial knowledge of the subject.

No. 1, contains only the five fundamental rules of Arithmetick. These rules have been treated of more largely than is customary, from the belief that most pupils pass from these to the more difficult rules before they are thoroughly acquainted with them. Fractions, and the Compound Rules, are entirely omitted in No. 1, until the learner is well acquainted with the working of whole numbers. No. 1, is also made small that the young learner may not be disheartened by having a large volume put into his hands; and that the parent shall not be under the necessity of purchasing a larger and more expensive book for his child before he shall require it.

No. 2, commences with the Compound Rules, and includes

all that is necessary of every other rule in Arithmetick for practical purposes, and the transactions of business. In No. 2, the EXPLANATIONS of the nature and principles of each rule are also fully and minutely given. No. 2, likewise, contains a Practical System of Book-Keeping. Tradesmen, without number, the most industrious and meritorious of men, often carry on their business with great difficulty, and many of them become involved and ruined, merely from the want of a simple system of keeping their accounts. Such a system is, therefore, given at the close of No. 2.

One very important advantage of this work is, that all, or nearly all, the questions for *practical* exercise are in dollars and cents.

The Author of the following work appeals, without apprehension or reluctance, to that publick, whose candour and liberality he has often experienced, to decide upon this attempt to render the elementary rules of Arithmetick both practical and popular, and also beneficial to the youth of this country.

LYMAN COBB.

New York, Jan. 25, 1832.

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EXPLANATION OF CHARACTERS USED IN THIS WORK.

$+$ Sign of addition.
$-$ Sign of substraction.
$\times$ Sign of multiplication.
$\div$ or) (Sign of Division.
$=$ Sign of equality.
: :: : Sign of proportion, thus, 4 : 8 :: 12 : 24, that is, as 4 is to 8, so is 12 to 24.
$\surd$ Radix, root, or side of a square.

COBB'S

EXPLANATORY ARITHMETICK.

Ques. What is Compound Arithmetick?

Ans. It is the working of figures employed to express quantities of different denominations.

Note.—To Teachers. All the explanations should be thoroughly and carefully read by the young scholar, as they are intended to impress deeply on his mind the principles of the rules, and their importance in his operations.

EXPLANATIONS.

You have already learned the five fundamental rules of Arithmetick. You have learned, to enumerate, to add, to substract, to multiply, and to divide numbers; that is, *simple*, *whole* numbers. You must now learn to work figures employed to express quantities of different denominations; as, bushels, pecks, quarts, and pints; pounds, ounces, and drachms; leagues, miles, and furlongs; yards, feet, and inches; years, months, and days; and for describing things of different values; that is, money of various sorts; as, eagles, dollars, dimes, cents, and mills. To work figures employed in these most useful purposes, is the present object of your attention. As the quantities and values are so various, and as several of them are occasionally joined to-

gether in one sum, or are *compounded*, the working of figures thus employed, is called COMPOUND ARITHMETICK.

You must not be alarmed, or apprehend any difficulty, in consequence of the extent or apparent intricacy of this branch of the study; for the principles in compound quantities do not materially differ from those in simple numbers, such as you have been working. You have, I presume, found it easy to express and to manage sums of the largest amount, by making yourself acquainted with the principles on which they are stated, and by which they are worked; and by a similar attention to a very simple rule or two, you will, also, see every difficulty vanish here; and will find it easy to work figures, however new and strange to you, the several quantities and values they may be employed to express.

If there had been only one unit for each kind of quantity, or if there had been different units increasing or decreasing in a tenfold proportion, then all Arithmetical operations, on the values of quantities, might have been expressed by the common or simple rules of Arithmetick. But for the sake of practical convenience, and from other causes, different units or denominations, for the same kind of quantity, and increasing in various proportions, have been introduced; and it is necessary that you should be acquainted with these units and their proportions; and that certain rules should be given for the convenient calculation of quantities, when represented in their various units or denominations.

Before you commence any operation in these compound rules, it is highly important, and, indeed, it is absolutely necessary, that you thoroughly learn the following Arithmetical tables.

FEDERAL MONEY.

This money increases in a tenfold proportion, and accounts are generally kept in it throughout the United States.

The denominations are, *Eagle*, *Dollar*, *Dime*, *Cent*, and *Mill*.

10 mills, *m.*	make	1 cent, *C.*
10 cents	-	1 dime, *D.*
10 dimes, or 100 cents,	-	1 dollar, $
10 dollars	-	1 eagle, *E.*

ENGLISH MONEY.

The denominations of this money are, *Pound*, *Shilling*, *Penny*, and *Farthing*.

4 farthings *qr.*	make	1 penny, *d.*
12 pence	-	1 shilling, *s.*
20 shillings	-	1 pound, £

TIME.

The denominations of time are, *Year*, *Month*, *Week*, *Day*, *Hour*, *Minute*, and *Second*.

60 seconds, *sec.*	make	1 minute, *min.*
60 minutes	-	1 hour, *H.*
24 hours	-	1 day, *D.*
7 days	-	1 week, *W.*
4 weeks	-	1 month, *mo.*
13 lunar months, 1 day and 6 hours, or 365 days, 6 hours		1 common or Julian year, *yr.*
12 calendar months		1 solar year, *yr.*
100 years	-	1 century, *C.*

AVOIRDUPOIS WEIGHT.

By this weight, all coarse and drossy goods, groceries, and all metals, except gold and silver, are weighed.

The denominations are, *Tun*, *Hundred-weight*, *Quarter*, *Pound*, *Ounce*, and *Drachm*.

16 drachms, *dr.*	make	1 ounce, *oz.*
16 ounces	-	1 pound, *lb.*
28 pounds	-	1 quarter, *qr.*
4 quarters	-	1 hundred-weight, *cwt.*
20 hundred-weight	-	1 tun, *T.*

APOTHECARIES WEIGHT.

This weight is used by apothecaries in compounding medicines; but all goods of this kind are bought and sold by Avoirdupois Weight.

The denominations are, *Pound*, *Ounce*, *Drachm*, *Scruple*, and *Grain*.

20 grains, *gr.*	make	1 scruple, ℈
3 scruples	-	1 drachm, ʒ
8 drachms	-	1 ounce, ℥
12 ounces	-	1 pound, ℔

TROY WEIGHT.

This weight is used for weighing gold, silver, jewellery, liquors, &c.

The denominations are, *Pound*, *Ounce*, *Penny-weight*, and *Grain*.

24 grains, *gr.*	make	1 penny-weight, *pwt.*
20 penny-weights	-	1 ounce, *oz.*
12 ounces	-	1 pound, *lb.*

DRY MEASURE.

This measure is used for grain, salt, coal, fruit, &c. The denominations are, *Chaldron*, *Bushel*, *Peck*, *Gallon*, *Quart*, and *Pint*.

A gallon in this measure contains $268\frac{4}{5}$ solid inches, and a bushel $2150\frac{42}{100}$.

2 pints, *pt.*	make	1 quart, *qt.*
4 quarts	- -	1 gallon, *gal.*
2 gallons	- -	1 peck, *p.*
4 pecks	- -	1 bushel, *bu.*
36 bushels	- -	1 chaldron of coal, *ch.*

WINE MEASURE.

This measure is applied to all spirituous liquors, vinegar, oil, &c.

The denominations are, *Tun*, *Pipe*, *Puncheon*, *Hogshead*, *Tierce*, *Barrel*, *Gallon*, *Quart*, *Pint*, and *Gill*.

4 gills, *gi.*	make	1 pint, *pt.*
2 pints	- - -	1 quart, *qt.*
4 quarts	- - -	1 gallon, *gal.*
$31\frac{1}{2}$ gallons	- - -	1 barrel, *bar.*
42 gallons	- - -	1 tierce, *tie.*
63 gallons	- - -	1 hogshead, *hhd.*
84 gallons	- - -	1 puncheon, *pun.*
2 hogsheads	- -	1 pipe, *P.*
2 pipes, or 4 hogsheads, or 252 gallons		1 tun, *T.*

The wine gallon contains 231 solid or cubick inches.

LONG MEASURE.

This measure is required where length is considered, without reference to breadth, as, for instance, the distance from one place to another.

The denominations are, *Degree*, *League*, *Mile*, *Furlong*, *Pole*, *Yard*, *Foot*, *Inch*, and *Barley-corn*.

3	barley-corns, *bc.*	make	1 inch, *in.*
12	inches - - -	-	1 foot, *ft.*
3	feet - - -	-	1 yard, *yd.*
$5\frac{1}{2}$	yards, or $16\frac{1}{2}$ feet -	-	1 pole or rod, *po.*
40	poles, or 220 yards	-	1 furlong, *fur.*
8	furlongs - - -	-	1 mile, *M.*
3	miles - - -	-	1 league, *L.*
60	geographick, or } miles	-	1 degree, *de.*
$69\frac{1}{2}$	statute }		
360	degrees, the circumference of the earth.		

A hand is 4 inches, and is used to measure the height of horses.

A chain is 4 rods, or 66 feet, and contains 100 links.

A fathom is 6 feet, and is chiefly used to measure the depth of water.

LAND OR SQUARE MEASURE.

This measure is used in ascertaining the contents of land, or of things which have length and breadth.

The denominations are, *Acre*, *Rood*, *Square Rod* or *Pole*, *Square Yard*, *Square Foot*, and *Square Inch*.

144	square	inches, *in.* make	1	square	foot, *S. F.*
9	-	feet - - -	1	-	yard, *yd.*
$30\frac{1}{4}$	-	yards, or $272\frac{1}{4}$ feet	1	-	pole, *po.*
40	-	poles - - -	1	-	rood, *R.*
4	-	roods - - -	1	-	acre, *A.*
640	-	acres - - -	1	-	mile, *M.*

SOLID OR CUBICK MEASURE.

This measure is used when things have length, breadth, and depth.

The denominations are, *Cord, Tun, Solid Yard, Solid Foot,* and *Solid Inch.*

1728 solid inches, *in.*	- make	1 solid foot, *ft.*
40 feet of round / 50 feet of hewn } timber	-	1 tun, *T.*
27 feet	- - - - -	1 solid yard, *yd.*
128 feet, or 8 feet long, 4 feet high, and 4 feet wide		1 cord, *C.*

A solid or cubick foot is 12 inches long, 12 broad, and 12 deep.

CLOTH MEASURE.

This measure is used for cloth, tapes, &c.

The denominations are, *Nail, Quarter, Yard,* the *Ell English, Flemish,* and *French.*

$2\frac{1}{4}$ inches, *in.*	make	1 nail, *na.*
4 nails	- - -	1 quarter, *qr.*
4 quarters	- - -	1 yard, *yd.*
3 quarters	- - -	1 Ell Flemish, *E. Fl.*
5 quarters	- - -	1 Ell English, *E. E.*
6 quarters	- - -	1 Ell French, *E. Fr.*

CIRCULAR MOTION.

This table is used by navigators, astronomers, &c. and relates to the heavenly bodies.

The denominations are, *Sign, Degree, Minute,* and *Second.*

60 seconds, ″	- make	1 minute, ′
60 minutes	- - -	1 degree, °
30 degrees	- - -	1 sign, *S.*
15 degrees of longitude	-	1 hour of time, *H.*

12 signs, or 360 degrees, the circle of the zodiack.

PAPER.

The denominations are, *Bale*, *Bundle*, *Ream*, *Quire*, and *Sheet*.

24 sheets, *s.*	make	1 quire, *q.*
20 quires	-	1 ream, *r.*
2 reams	-	1 bundle, *bun.*
10 reams	-	1 bale, *ba.*

The sizes of paper are various, and are usually denominated by stationers, pot, foolscap, post, crown, demy, medium, royal, super-royal, imperial, &c.

It is also usual to put 20 sheets in the two outside quires of each ream, which are broken and defective: these are called casse.

MISCELLANEOUS.

This table is particularly useful to stationers, book-dealers, &c.

12 particulars, or single things	make	1 dozen, *doz.*
5 dozen	-	1 roll, *R.*
12 dozen	-	1 gross, *G.*
12 gross	-	1 great gross, *G. G.*
20 single things	-	1 score, *S.*
5 score	-	1 hundred, *H.*
12 skins of parchment,	-	1 roll, *R.*

BOOKS.

Folio is the largest size of books, of which 2 leaves or 4 pages, make a sheet.

Quarto, 4to. 4 leaves, or 8 pages, make a sheet.

Octavo, 8vo. 8 leaves, or 16 pages, make a sheet.

Duodecimo, 12mo. 12 leaves, or 24 pages, make a sheet.

Octodecimo, 18mo. 18 leaves, or 36 pages, make a sheet.

COMPOUND ADDITION.

Q. What is Compound Addition?

A. Compound Addition teaches to join, or add, several numbers, or quantities, of different denominations into one sum.

EXAMPLES

For Mental Exercise.

1. If you pay two cents and five mills for one orange, and three cents and five mills for another; how many cents do you pay for both?

2. If you have five cents in one hand, and fifteen cents in the other; how many dimes have you in both?

3. James paid six cents and five mills for a primer, four cents for a top, and seven cents and five mills for an inkstand; how many cents did he pay for all?

4. John bought a coat for twelve dollars and twenty-five cents, and a hat for four dollars and seventy-five cents; how many dollars did he pay for both?

5. William had one dollar, five dimes, and five mills, and Thomas had two dollars, four dimes, and five mills; how many dollars had both of them?

6. George paid two shillings and six pence for one book, and three shillings and six pence for another; how many shillings did he pay for both?

7. Jane paid five shillings for cambrick, nine pence for riband, and three pence for thread; how many shillings did she pay for the whole?

8. Rufus bought a watch for six pounds and twelve shillings, and a pair of new boots for two pounds and eight shillings; how many pounds did he pay for both?

9. James spent at school one hour and fifteen minutes in the study of Arithmetick, forty-five minutes in writing, and one hour in studying grammar; how many hours did he spend in school?

10. William spent one day and ten hours in the city of New York, and two days and fourteen hours in Philadelphia; how many days did he spend in both places?

11. James bought twelve ounces of sugar plums, one pound and four ounces of raisins; how many pounds had he of both?

12. A gentleman bought at one store three quarters of sugar, and at another store two hundred-weight and one quarter; how many hundred-weight did he buy at both places?

13. James, William, and Thomas, went into the field to gather chestnuts. James gathered four quarts and one pint, William gathered six quarts, and Thomas gathered five quarts and one pint; how many pecks of chestnuts did they gather?

14. If you buy four bushels and one peck of wheat of one farmer, and five bushels and three pecks of another; how many bushels do you buy of both?

15. A gentleman had three quarts, one pint, and three gills of wine in one bottle, and one pint and one gill in another; how many gallons of wine had he in both bottles?

16. If you have one stick that is ten inches long, and another that is two feet and two inches long; how many feet long are both of them?

17. A lady bought one piece of cloth containing five yards, three quarters, and three nails, and another

containing six yards and one nail; how many yards did both pieces contain?

18. William bought two quires and sixteen sheets of paper at one store, and one quire and eight sheets of paper at another store; how many quires of paper did he buy at both places?

Note.—To Teachers. The learner should be required to answer, mentally, the preceding questions, and various others of an equally simple nature, before he is required to use a slate.

RULE.

Q. How must the different numbers, or quantities to be added, be placed in Compound Addition?

A. They must be placed so that the numbers of the same denomination will stand directly under each other.

Q. Where must you begin to add in Compound Addition?

A. At the right hand, or lowest denomination, as in Simple Addition.

Q. Why do you begin to add at the right hand denomination?

A. Because the different denominations increase in quantity from the right hand to the left, as in Simple Addition.

Q. How must the first column be added?

A. The same as in Simple Addition.

Q. What sum must you set down, and what must you carry to the next column in Compound Addition?

A. The amount must be divided by the number that it takes of that denomination to make one in the next higher denomination, and the remainder, if there be any, must be set down, and the quotient must be carried to the next higher denomination.

Q. How must the left hand, or highest denomination be added?

A. It must be added up, and the whole amount set down, as in Simple Addition.

EXAMPLES

For Theoretical Exercise on a Slate.

FEDERAL MONEY.

1. William paid 1 dime, 2 cents, and 5 mills for a spelling-book, 3 dimes and 1 cent for an arithmetick, 2 dimes and 5 cents for a slate, 4 dimes and 5 cents for a geography, and 7 dimes and 5 cents for a dictionary; how much did he pay for all of them? *Ans.* 1 dollar, 8 dimes, 8 cents, and 5 mills.

EXPLANATIONS.

Federal money increases in a tenfold proportion, and is, in its simplicity, nearly allied to whole numbers; and, consequently, the rule which you used in adding whole numbers, may be used. You will remember, that the first thing to be done is to place the different denominations directly under each other; as, dollars under dollars dimes under

d.	*c.*	*m.*
1,	2,	5
3,	1,	0
2,	5,	0
4,	5,	0
7,	5,	0
$1,8,	8,	5

dimes, cents under cents, and mills under mills. You must place a comma between the dollars, dimes, cents, and mills, then add as in whole numbers. If your sum consists of dollars and mills only, or of dollars and cents only, you must place a cipher in the vacant place, as in the present example. Accounts are generally kept, however, in dollars, cents, and mills, without a comma being used between the dimes and cents. Beginning at the right hand column, you must say, 5 is five, that is, five mills; you must set down the 5 in the place of mills. As there is nothing to be carried to the next column, you must begin anew with the 5 in the place of cents, and say, 5 and 5 are ten, and 5 make fifteen, 1 makes sixteen, and 2 make eighteen cents, that is, one dime and eight cents; you must set down the 8 under the column of cents, and add, or carry, the one dime to the next column, the place of dimes. Thus, one dime added to the 7, in the column of dimes, make eight, 4 make twelve, 2 make fourteen, 3 make seventeen, and 1 make eighteen dimes, that is, one dollar and eight dimes; you must set down the 8 under the column of dimes, and carry the one dollar to the next column, the place of dollars, which makes the whole amount 1 dollar, 8 dimes, 8 cents, and 5 mills.

PROOF.

The methods of proof are the same in Compound Addition as in Simple Addition.

2. James bought a suit of clothes for 25 dollars and 75 cents, a hat for 7 dollars and 25 cents, a pair of boots for 6 dollars, and a watch for 18 dollars, 37 cents, and 5 mills; how much did he pay for the whole? *Ans.* \$57,37,5.

EXPLANATIONS.

$	c.	m.
25,	75,	0
7,	25,	0
6,	00,	0
18,	37,	5
$57,	37,	5

You will perceive that, in this example, I have placed the comma between dollars, cents, and mills only, as accounts are thus generally kept, without reference to dimes. You must begin, as before, with the column at the right hand, and say, 5 is five, that is, five mills; you must set down the 5 in the place of mills. As there is nothing to be carried to the next column, you must begin anew with the 7, in the place of cents, and say, 7 and 5 are twelve, and 5 make seventeen cents; you must set down the 7 under the right hand column of cents, and add, or carry, one to the next column, the tens of cents. Thus, one ten added to the 3, in the next column, make four, 2 make six, and 7 make thirteen dimes, or tens of cents, that is, one dollar and thirty cents; you must set down the 3 under the second column of cents, or place of dimes, and add, or carry, the one to the next column, the place of dollars. Thus, one dollar added to the 8, in the next column, make nine, 6 make fifteen, 7 make twenty-two, and 5 make twenty-seven dollars, that is, seven units of dollars, and two tens of dollars; you must set down the 7 in the first column of dollars, and add, or carry, two to the next column, the place of tens of dollars. Thus, two tens of dollars added to the 1, in the second column of dollars, make three, and 2 make five, that is, five tens of dollars; you must set down the 5 in the second column of dollars. Thus you have fifty-seven dollars, thirty-seven cents, and five mills, and your work is done.

By paying particular attention to the use of the

comma, in separating the dollars, cents, and mills, you will be able to work any sum in Addition of federal money; for, indeed, the proper placing of the comma is the only operation which distinguishes this from Simple Addition. I shall, therefore, give you a few examples for farther exercise, and pass to the next part of this subject.

(3.)	(4.)	(5.)
\$ c. m.	\$ c. m.	\$ c. m.
8, 46, 4	37, 56, 1	736, 36, 1
9, 58, 3	12, 81, 0	481, 61, 4
1, 67, 7	61, 16, 3	218, 63, 8
6, 33, 8	78, 18, 6	108, 40, 3
9, 47, 2	92, 24, 7	296, 84, 6
\$35, 53, 4		

(6.)	(7.)	(8.)
\$ c. m.	\$ c. m.	\$ c. m.
2, 60, 3	12, 34, 5	271, 35, 6
5, 82, 1	98, 76, 5	311, 96, 4
3, 74, 2	44, 44, 3	964, 55, 2
1, 15, 6	66, 55, 4	813, 36, 6
1, 00, 8	77, 66, 4	535, 75, 4
9, 87, 3	38, 21, 3	749, 46. 9

STERLING OR ENGLISH MONEY.

1. A man bought a firkin of butter for £6 9*s.* 7*d.* 2*qr.*, a barrel of pork for £8 7*s.* 8*d.*, a cag of molasses for £2 5*s.* 6*d.* 3*qr.*, and a barrel of flour for £3 10*s.*; how much did he pay for the whole? *Ans.* £20 12*s.* 10*d.* 1*qr.*

EXPLANATIONS.

£	s.	d.	qr.
6	9	7	2
8	7	8	0
2	5	6	3
3	10	0	0
£20	12	10	1

As before, you have first to place the same denominations directly under each other; as, farthings under farthings, pence under pence, &c. Beginning with the lowest denomination, the farthings, you must say, 3 and 2 are five, that is, five farthings; you must divide the amount, the 5, by 4, because four farthings are equal to, or make one penny, and you will find that the quotient will be one, and the remainder 1; and you must set down the 1, the remainder, under the column of farthings, and add, or carry, the one, the quotient, to the next column, the pence, the next higher denomination, because four farthings are equal to one penny. Thus, one penny added to the 6, in the column of pence, makes seven, 8 make fifteen, and 7 make twenty-two, that is, twenty-two pence; you must divide the amount, the 22, by 12, because twelve pence are equal to, or make one shilling, and the quotient will be one, and the remainder 10; and you must set down the 10, the remainder, under the column of pence, and add, or carry, the one, the quotient, to the next column, the shillings, the next higher denomination, because twelve pence are equal to one shilling. Thus, one shilling added to the 10, in the column of shillings, makes eleven, 5 make sixteen, 7 make twenty-three, and 9 make thirty-two, that is, thirty-two shillings; you must divide the amount, the 32, by 20, because twenty shillings are equal to, or make one pound, and the quotient will be one, and the remainder 12; and you must set down the 12, the remainder, under the column of

shillings, and add, or carry, the one, the quotient, to the next column, the pounds, the next higher denomination, because twenty shillings are equal to one pound. Thus, one pound added to the 3, in the column of pounds, make four, 2 make six, 8 make fourteen, and 6 make twenty, that is, twenty pounds; and you must set down the whole number 20, as pound is the highest, or largest denomination in sterling money. You must add the column of pounds, as in Simple Addition, and set down the whole amount, and then the work is done.

In order to pursue this new course with advantage, it will be necessary to attend to the sort of notation which is used in Compound Arithmetick; and, indeed, this is nearly all that you have now to learn, if you have learned the Arithmetical Tables thoroughly, to be able to perform every operation in this branch of Arithmetick.

In the notation of which I have before treated, you will remember, that every advance in the value, or in the station of figures, is by tens, that is, in that notation we count from one to nine, and the next higher number is expressed by one with a cipher after it. In short, in the former part of Arithmetick, in that which we call *Simple* Arithmetick, in contradistinction to this, which we call *Compound*, we count by tallies of tens; and, indeed, on this principle is all numeration, all notation, and all calculation, carried on in Simple Arithmetick.

But when we come to Compound Arithmetick, as, for instance, in counting sterling money, we begin with farthings, and as soon as we have four, (*not ten,*) we come to one, that is, one penny: then we count on to the next denomination, which is twelve pennies, and we call one, that is, one shilling: then,

again, we count on to twenty, when it becomes one again, that is, one pound. Thus, in the notation of sterling money, instead of counting on to ten, and then changing, we count first to four, then to twelve, and then to twenty. If we wish to count by our measure of inches, feet, yards, &c., we count first twelve inches make one foot, then, three feet make one yard, and so on. If we wish to count by our ordinary weights, we count sixteen, twenty-eight, four, and twenty, that is, sixteen ounces make one pound, twenty-eight pounds make one quarter of a hundred-weight, four quarters make one hundred-weight, and twenty hundred-weight make one tun, and so on.

Thus you will perceive, that each sort of money, each sort of weight, and each sort of measure, has its peculiar notation; and this peculiarity must be known, and kept in mind, when you work figures descriptive of any of these several quantities.

I must also inform you, that you must divide, or separate, by means of a dot or two, a comma, or a space, the figures descriptive of the different sorts of money; and that, in order to show that certain figures are employed to describe pounds in sterling money, we write before, or over it, this character £; that over figures descriptive of shillings, we write a small *s;* over those for pence, we write a small *d;* and over farthings, we write *qr.*

This description of the notation of sterling money, contains, and illustrates the principle on which the notation of all money, of whatever country, of all measures, and of all weights, is conducted.

The PRINCIPLE of this is very plain. The POUND STERLING is the *whole;* the SHILLINGS are regarded merely as *parts* of the POUND, that is, *twentieths;*

the PENNIES the *twelfths* of the shilling; and FARTHINGS the *fourths* of the penny. Hence it is, that when you have twelve pennies, you call them a shilling; and when you have reckoned up twenty shillings you call them a pound: but the pound being the WHOLE, you count pounds, and reckon them as I have just stated, as you learned to treat whole numbers in Simple Arithmetick; and thus it is to be in all your reckonings, that is, when you deal with the highest denomination, whether of money, of weights, or of measures, you must treat it as you would treat the same sum in Simple Arithmetick.

Whenever, therefore, you have to state, or to reckon any sort of money, weights, or measures, the principle on which you have to proceed is this, to ascertain what is the whole number, and what are the parts into which the money, the weights, or the measures, of which you have to treat, have been divided; and then you will know how to proceed. You have been told how to proceed in the notation and addition of sterling money.

We will take an example of the treatment of our larger WEIGHTS. The TUN is the largest weight to which we reckon; it is, therefore, the whole number, and it is broken down, or divided, into HUNDRED-WEIGHTS, twenty of which make one tun. The hundred-weight being composed of a hundred and twelve pounds, a sum inconveniently large, it is divided into QUARTERS, of twenty-eight pounds each; and for still greater convenience, the POUND is divided into sixteen ounces, so that the numeration of our larger weights is, sixteen ounces make one pound, twenty-eight pounds make one quarter, four quarters make one hundred-weight, and twenty hundred-weight make one tun. The marks used to describe

these several weights, are *T.*, *cwt.*, *qr.*, *lb.*, *oz.*, as you have already learned in the Arithmetical Tables.

The several sorts of weights and measures are differently divided, and have names varying from each other. But you must not startle at the multiplicity of these things; for you must remember, that you are furnished with the *principle* on which they are all to be treated and managed. You have seen, that all you have to do, on proceeding to work figures descriptive of any of them, is to know, first, what is its whole number, or, as it is commonly called, its largest denomination, and then to know how that is divided. These things are all stated in the Arithmetical Tables. As before stated, the principles of Compound Arithmetick do not materially differ from those of Simple Arithmetick. Thus, in Simple Arithmetick you carry one for every ten; but in Compound Arithmetick you carry by the number that it takes to make, or equal one in the next higher denomination; this is all the difference.

By paying particular attention to the preceding EXPLANATIONS, you will be able to work any sum in Compound Addition; and I wish you to bear in mind continually, that the object of these EXPLANATIONS is to explain to you the principles upon which the rules are founded.

As sterling money is but very little used in this country, I shall merely give you a few sums for exercise, and pass to the next part of the subject.

NOTE.—TO TEACHERS. The teacher should be very particular in requiring the learner to place the figures in each column with great accuracy; as, farthings under farthings, pence under pence; ounces under ounces, pounds under pounds; pints under pints, quarts under quarts, &c.; for neglect in this particular would very naturally lead him into errour in performing the operation.

(2.)				(3.)				(4.)			
£	s.	d.	qr.	£	s.	d.	qr.	£	s.	d.	qr.
36	10	6	3	142	8	7	2	189	12	8	0
98	14	4	1	963	17	8	1	233	9	10	3
76	13	11	2	147	13	10	3	199	15	7	2
11	9	5	0	329	4	7	2	467	16	11	1
61	7	0	3	732	11	6	1	823	8	4	2

TIME.

1. Add together 28*yr.* 123*d.* 14*h.* 31*min.* 15*sec.*, 19*yr.* 14*d.* 13*h.* 55*min.* 56*sec.*, 119*yr.* 176*d.* 23*h.* 28*min.* 36*sec.*, and 232*yr.* 114*d.* 17*h.* 34 *min.* 59*sec.* *Ans.* 399*yr.* 64*d.* 21*h.* 30*min.* 46*sec.*

EXPLANATIONS.

yr.	*d.*	*h.*	*min.*	*sec.*
28	123	14	31	15
19	14	13	55	56
119	176	23	28	36
232	114	17	34	59
399	64	21	30	46

In this example, you must begin as before with the lowest denomination, at the right hand, the seconds. Beginning with the right hand column of seconds, you must say, 9 and 6 are fifteen, 6 make twenty-one, and 5 make twenty-six, that is, twenty-six units of seconds; you must set down the 6 in some convenient place on the slate, or board, and add, or carry, the two tens to the second column of seconds. Thus, two tens of seconds added to the 5, in the second column of seconds, make seven, 3 make ten, 5 make fifteen, and 1 makes sixteen, that is, sixteen tens of seconds; and this being the last column of seconds, you must set down the 16 at the left of the 6, the product of the first column of seconds, and then you will have the whole amount

of the seconds, which is one hundred and sixty-six; you must divide the 166 by 60, because sixty seconds make, or are equal to one minute; and you will find the quotient two, and the remainder 46; you must set down the remainder, the 46, under the column of seconds, and add, or carry, the two, the quotient, to the column of minutes, the next higher denomination, because sixty seconds are equal to one minute. Thus, two minutes added to the 4, in the right hand column of minutes, make six, 8 make fourteen, 5 make nineteen, and 1 makes twenty, that is, twenty units of minutes; you must set down the cipher in some convenient place, as before directed, and add, or carry, the two tens to the second column of minutes. Thus, two tens of minutes, added to the 3, in the second column of minutes, make five, 2 make seven, 5 make twelve, and 3 make fifteen, that is, fifteen tens of minutes; and this being the second, or last column of minutes, you must set down the 15 at the left of the cipher, the product of the first column of minutes, and then you will have the whole amount of the minutes, which is one hundred and fifty; you must divide the 150 by 60, because sixty minutes make, or are equal to one hour, and you will find the quotient two, and the remainder 30; you must set down the remainder, the 30, under the column of minutes, and add, or carry, the two, the quotient, to the column of hours, the next higher denomination, because sixty minutes are equal to one hour. Thus, two hours added to the 7, in the right hand column of hours, make nine, 3 make twelve, 3 make fifteen, and 4 make nineteen, that is, nineteen units of hours; you must set down the 9 in some convenient place, as before directed, and add, or carry, the one ten to the second column of hours.

Thus, one ten of hours added to the 1, in the second column of hours, makes two, 2 make four, 1 makes five, and 1 makes six, that is, six tens of hours; and this being the second, or last column of hours, you must set down the 6 at the left of the 9, the product of the first column of hours, and then you will have the whole amount of the hours, which is sixty-nine; you must divide the 69 by 24, because twenty-four hours make, or are equal to one day, and you will find the quotient two, and the remainder 21; you must set down the remainder, the 21, under the column of hours, and add, or carry, the two, the quotient, to the column of days, the next higher denomination, because twenty-four hours are equal to one day. Thus, two days added to the 4, in the right hand column of days, make six, 6 make twelve, 4 make sixteen, and 3 make nineteen, that is, nineteen units of days; you must set down the 9 in some convenient place, and add, or carry, the one ten to the second column of days. Thus, one ten of days added to the 1, in the second column of days, makes two, 7 make nine, 1 makes ten, and 2 make twelve, that is, twelve tens, or one hundred and two tens of days; and you must set down 2 at the left of the 9, the product of the first column of days, and add, or carry, the one hundred to the third column of days. Thus, one hundred added to the 1, in the third column of days, makes two, 1 makes 3, and 1 makes four, that is, four hundreds of days; and this being the last column of days, you must set down the 4 at the left of the 29, the product of the first and second column of days, and then you will have the whole amount of the days, which is four hundred and twenty-nine; you must divide the 429 by 365, because 365 days make, or are equal to one year, and

you will find the quotient one, and the remainder 64; you must set down the remainder, the 64, under the column of days, and add, or carry, the one, the quotient, to the column of years, the next higher denomination, because three hundred and sixty-five days are equal to one year. As the year is the highest denomination of time, you must add the columns of years as you added in Simple Arithmetick, and set down the whole amount, and then the work is done.

You must always remember, that when the amount of an inferiour, or lower denomination does not amount to, or equal one of the next higher denomination, the whole must be set down, and that nothing is then to be carried to the next denomination.

I wish you to read the preceding EXPLANATIONS with great care and attention, as your progress will, I trust, be greatly accelerated by so doing.

(2.)

yr.	*mo.*	*w.*	*d.*	*h.*	*min.*	*sec.*
12	10	2	4	19	34	56
96	3	1	5	21	13	29
14	11	3	6	17	37	42
84	2	1	3	18	53	47
36	7	1	2	14	51	37

(3.)

yr.	*d.*	*h.*	*min.*	*sec.*
24	124	13	51	52
92	63	15	44	46
36	128	12	19	10
144	10	18	19	20
77	96	13	13	9

AVOIRDUPOIS WEIGHT.

1. A merchant bought 13*T.* 3*cwt.* 4*lb.* 9*oz.* of pearlash, 1*T.* 19*cwt.* 3*qr.* 8*lb.* 6*oz.* of cheese, 17*cwt.* 2*qr.* 4*oz.* 14*dr.* of rice, and 16*cwt.* 1*qr.* 19*lb.* 11*oz.* 8*dr.* of coffee; what was the weight of the whole? *Ans.* 16*T.* 16*cwt.* 3*qr.* 4*lb.* 15*oz.* 6*dr.*

EXPLANATIONS.

T.	cwt.	qr.	lb.	oz.	dr.
13	3	0	4	9	0
1	19	3	8	6	0
	17	2	0	4	14
	16	1	19	11	8
16	16	3	4	15	6

In this example, beginning, as before, with the right hand denomination, you must add up the drachms and find the amount; and then divide that amount by 16, because sixteen drachms are equal to one ounce; set down the remainder, which will be drachms, under the column of drachms, and add, or carry, the quotient, which will be ounces, to the next column, the column of ounces. Add up the column of ounces and find the amount; then divide that amount by 16. because sixteen ounces are equal to one pound; set down the remainder, which will be ounces, under the column of ounces, and add, or carry, the quotient, which will be pounds, to the next column, the column of pounds. Add up the column of pounds and find the amount; then divide that amount by 28, because twenty-eight pounds are equal to one quarter; set down the remainder, which will be pounds, under the column of pounds, and add, or carry, the quotient, which will be quarters, to the next column, the column of quarters. Add up the column of quarters and find the amount; then divide that amount by 4, because four quarters are equal to one hundred-weight; set down the remainder, which will be quarters, under the column of quarters, and add, or carry, the quotient, which will be hundred-weight, to the next column, the column of hundred-weight. Add up the column of hundred-weight and find the amount; then divide that amount by 20, because twenty hundred-weight are equal to one tun; set down the

remainder, which will be hundred-weight, under the column of hundred-weight, and add, or carry, the quotient, which will be tuns, to the next column, the column of tuns. Add up the tuns, and set down the whole amount, as in Simple Addition, as tun is the highest denomination in avoirdupois weight.

(2.)

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
77	19	3	27	15	11
96	6	3	2	2	9
74	11	2	19	14	5
41	13	2	17	10	12
19	17	1	24	9	7

(3.)

cwt.	*qr.*	*lb.*	*oz.*	*dr.*
5	0	8	9	7
17	2	2	3	8
10	1	19	13	12
19	2	23	15	9
9	1	11	10	14

APOTHECARIES WEIGHT.

1. Add together 27℔ 10℥ 1ʒ 2℈ 18*gr.*, 17℔ 9℥ 7ʒ 1℈ 14*gr.*, 24℔ 0℥ 0ʒ 0℈ 5*gr.*, and 147℔ 4℥ 4ʒ 1℈ 13*gr.* *Ans.* 217℔ 0℥ 6ʒ 0℈ 10*gr.*

EXPLANATIONS.

℔	℥	ʒ	℈	*gr.*
27	10	1	2	18
17	9	7	1	14
24	0	0	0	5
147	4	4	1	13
217	0	6	0	10

Begin, as before, at the right hand, or lowest denomination, and add the grains; divide the amount by 20, because twenty grains are equal to one scruple; set down the remainder, and carry the quotient to the column of scruples. Add the scruples; divide the amount by 3, because three scruples are equal to one drachm; set down the remainder, but in this example there is no remainder, therefore, you must set down

a cipher, and carry the quotient to the column of drachms. Add the drachms; divide the amount by 8, because eight drachms are equal to one ounce; set down the remainder, and carry the quotient to the column of ounces. Add the ounces; divide the amount by 12, because twelve ounces are equal to one pound; set down a cipher in this example, under the column of ounces, as there is no remainder, and carry the quotient to the right hand column of pounds. Add the pounds, and set down the whole amount, as in Simple Addition, because pound is the highest denomination in apothecaries weight.

(2.)

℔	℥	ʒ	℈	*gr.*
57	6	3	2	16
42	5	1	0	19
10	4	5	1	12
13	11	6	2	7
71	3	7	1	12

(3.)

℔	℥	ʒ	℈	*gr.*
9	9	2	1	6
12	0	0	2	10
41	8	7	1	19
6	10	1	0	12
18	11	6	2	15

TROY WEIGHT.

1. Add together 27*lb.* 10*oz.* 17*pwt.* 8*gr.*, 117*lb.* 9*oz.* 12*pwt.* 14*gr.*, 133*lb.* 6*oz.* 13*pwt.* 15*gr.*, 220*lb.* 23*gr.* *Ans.* 499*lb.* 3*oz.* 4*pwt.* 12*gr.*

EXPLANATIONS.

Begin, as before, at the right hand, or lowest denomination, and add the grains; divide the amount by 24, because twenty-four grains are equal to one penny-weight; set down the remainder, and carry the quotient to the column of penny-

lb.	*oz.*	*pwt.*	*gr.*
27	10	17	8
117	9	12	14
133	6	13	15
220	0	0	23
499	3	4	12

weights. Add the penny-weights; divide the amount by 20, because twenty penny-weights are equal to one ounce; set down the remainder, and carry the quotient to the column of ounces. Add the ounces; divide the amount by 12, because twelve ounces are equal to one pound; set down the remainder, and carry the quotient to the right hand column of pounds. Add the pounds, and set down the whole amount, as in Simple Addition, because pound is the highest denomination in troy weight.

(2.)

lb.	*oz.*	*pwt.*	*gr.*
12	9	19	21
2	8	10	16
9	10	12	14
10	4	13	19
16	11	14	11

(3.)

lb.	*oz.*	*pwt.*	*gr.*
96	7	16	22
111	11	18	15
36	3	4	8
47	10	13	12
258	9	11	10

DRY MEASURE.

1. A farmer had 31*bu.* 3*p.* 4*qt.* of wheat in one bin, 90*bu.* 6*qt.* in another bin, 22*bu.* 2*p.* 7*qt.* 1*pt.* in another, and 11*bu.* 1*p.* 5*qt.* 1*pt.* in another; how many bushels had he in all? *Ans.* 156*bu.* 0*p.* 7*qt.* 0*pt.*

EXPLANATIONS.

bu.	*p.*	*qt.*	*pt.*
31	3	4	0
90	0	6	0
22	2	7	1
11	1	5	1
156	0	7	0

Begin, as before, at the right hand denomination, and add the pints; divide the amount by 2, because two pints are equal to one quart; set down the remainder, but in this example there is no remainder to be set down under the pints, you must, therefore, set down a

cipher, and carry the quotient to the column of quarts. Add the quarts; divide the amount by 8, because eight quarts are equal to one peck; set down the remainder, and carry the quotient to the column of pecks. Add the pecks; divide the amount by 4, because four pecks are equal to one bushel; set down the remainder, but in this example there is no remainder, therefore, you must set down a cipher under the column of pecks, and carry the quotient to the right hand column of bushels. Add the bushels, and set down the whole amount, as in Simple Addition, because bushel is the highest denomination in dry measure, in the measure of wheat, &c., as in this example.

(2.)

bu.	*p.*	*qt.*	*pt.*
23	2	7	1
17	3	6	0
16	1	4	1
27	0	5	0
13	2	4	1

(3.)

ch.	*bu.*	*p.*	*qt.*	*pt.*
28	24	3	7	1
7	28	2	3	1
73	14	3	5	1
64	13	0	4	0
52	21	2	6	1

WINE MEASURE.

1. Add together 13*T.* 1*p.* 1*hhd.* 54*gal.* 3*qt.* 1*pt.* 3*gi.*, 136*T.* 1*p.* 24*gal.* 2*qt.* 1*gi.*, 11*T.* 1*p.* 1*hhd.* 19*gal.* 3*qt.* 1*pt.* 1*gi.*, and 12*T.* 1*p.* 1*hhd.* 57*gal.* 3*qt.* 1*pt.* 3*gi.*
Ans. 175*T.* 0*p.* 1*hhd.* 31*gal.* 1*qt.* 1*pt.* 0*gi.*

EXPLANATIONS.

T.	p.	hhd.	gal.	qt.	pt.	gi.
13	1	1	54	3	1	3
136	1	0	24	2	0	1
11	1	1	19	3	1	1
12	1	1	57	3	1	3
175	0	1	31	1	1	0

Begin, as before, at the right hand denomination, and add the gills; divide the amount by 4, because four gills are equal to one pint; set down the remainder, or in this example a cipher, as there is no remainder, and carry the quotient to the column of pints. Add the pints; divide the amount by 2, because two pints are equal to one quart; set down the remainder, and carry the quotient to the column of quarts. Add the quarts; divide the amount by 4, because four quarts are equal to one gallon; set down the remainder, and carry the quotient to the column of gallons. Add the gallons; divide the amount by 63, because 63 gallons are equal to one hogshead; set down the remainder, and carry the quotient to the column of hogsheads. Add the hogsheads; divide the amount by 2, because two hogsheads are equal to one pipe; set down the remainder, and carry the quotient to the column of pipes. Add the pipes; divide the amount by 2, because two pipes are equal to one tun; set down the remainder, or in this example a cipher, as there is no remainder, and carry the quotient to the column of tuns. Add the tuns, and set down the whole amount, as in Simple Addition, because tun is the highest denomination in wine measure.

(2.)				
bar.	*gal.*	*qt.*	*pt.*	*gi.*
5	17	3	1	3
27	25	1	0	2
61	19	2	1	1
84	29	3	0	3
73	14	0	1	2

(3.)				
hhd.	*gal.*	*qt.*	*pt.*	*gi.*
16	24	3	0	1
119	57	1	1	3
36	39	2	0	2
240	9	0	1	1
25	61	2	1	2

LONG MEASURE.

1. Add together 10*de.* 10*m.* 5*fur.* 24*rd.* 4*yd.* 2*ft.* 9*in.* 2*bc.*, 56*de.* 54*m.* 7*fur.* 35*rd.* 3*yd.* 1*ft.* 8*in.* 1*bc.*, 36*de.* 43*m.* 4*fur.* 11*rd.* 4*yd.* 1*ft.* 11*in.* 1*bc.*, and 79*de.* 57*m.* 6*fur.* 16*rd.* 5*in.* *Ans.* 183*de.* 47*m.* 0*fur.* 8*rd.* 2*yd.* 0*ft.* 10*in.* 1*bc.*

EXPLANATIONS.

de.	*m.*	*fur.*	*rd.*	*yd.*	*ft.*	*in.*	*bc.*
10	10	5	24	4	2	9	2
56	54	7	35	3	1	8	1
36	43	4	11	4	1	11	1
79	57	6	16	0	0	5	0
183	47	0	8	2	0	10	1

Begin, as before, at the right hand column, and add the barley-corns; divide the amount by 3, because three barley-corns are equal to one inch; set down the remainder, and carry the quotient to the column of inches. Add the inches; divide the amount by 12, because twelve inches are equal to one foot; set down the remainder, and carry the quotient to the column of feet. Add the feet; divide the amount by 3, because three feet are equal to one yard; set down the remainder, but in this example there is no remainder, therefore, you must set down a cipher, and carry the quotient to the

column of yards. Add the yards; divide the amount by $5\frac{1}{2}$, because five and a half yards are equal to one rod; set down the remainder, and carry the quotient to the column of rods. Add the rods; divide the amount by 40, because forty rods are equal to one furlong; set down the remainder, and carry the quotient to the column of furlongs. Add the furlongs; divide the amount by 8, because eight furlongs are equal to one mile; set down the remainder, but in this example there is no remainder, therefore, you must set down a cipher, and carry the quotient to the column of miles. Add the miles; divide the amount by 60, because 60 miles are equal to one geographick de.; set down the remainder, and carry the quotient to the column of degrees. Add the degrees, and set down the whole amount, as in Simple Addition, because degree is the highest denomination in long measure.

(2.)

yd.	*ft.*	*in.*	*bc.*
36	0	7	1
29	1	8	2
27	2	5	2
35	2	9	1
28	1	7	2

(3.)

m.	*fur.*	*rd.*	*yd.*	*ft.*	*in.*	*bc.*
136	3	29	4	1	10	1
678	5	38	2	2	9	2
487	6	36	3	1	8	1
643	7	28	4	2	10	2
786	6	30	5	1	11	1

LAND OR SQUARE MEASURE.

1. Add together 219*m.* 150*a.* 3*r.* 38*po.* 5*yd.* 5*ft.* 14*in.*, 1167*m.* 320*a.* 2*r.* 30*po.* 0*yd.* 6*ft.* 140*in.*, 237*m.* 420*a.* 3*r.* 36*po.* 16*yd.* 8*ft.* 134*in.*, and 431*m.* 136*a.* 2*r.* 20*po.* 4*yd.* 3*ft.* 116*in.* *Ans.* 2055*m.* 389*a.* 1*r.* 4*po.* 27*yd.* 6*ft.* 116*in.*

EXPLANATIONS.

m.	a.	r.	po.	yd.	ft.	in.
219	150	3	38	5	5	14
1167	320	2	30	0	6	140
237	420	3	36	16	8	134
431	136	2	20	4	3	116
2055	389	1	4	27	6	116

Begin, as before, at the right hand column, and add the inches; divide the amount by 144, because one hundred and forty-four square inches are equal to one square foot; set down the remainder, and carry the quotient to the column of feet. Add the feet; divide the amount by 9, because nine square feet are equal to one square yard; set down the remainder, and carry the quotient to the column of yards. Add the yards; divide the amount by $30\frac{1}{4}$, because thirty and a quarter square yards are equal to one square pole, but in this example the whole number of yards does not amount to thirty and a quarter, therefore, you must set it down, and there will be nothing to be carried to the column of poles. Add the poles; divide the amount by 40, because forty square poles are equal to one square rood; set down the remainder, and carry the quotient to the column of roods. Add the roods; divide the amount by 4, because four roods are equal to one acre; set down the remainder, and carry the quotient to the column of acres. Add the acres; divide the amount by 640, because six hundred and forty square acres are equal to one square mile; set down the remainder, and carry the quotient to the column of miles. Add the miles, and set down the whole amount, as in Simple Addition, because mile is the highest denomination in land or square measure.

(2.)

a.	r.	po.	yd.	ft.	in.
245	3	14	29	3	143
36	3	27	30	7	137
224	1	39	28	2	141
275	3	16	27	3	127
362	2	2	26	1	136

(3.)

a.	r.	po.
195	1	24
267	2	38
437	3	29
678	2	36
789	3	28

SOLID OR CUBICK MEASURE.

1. Add together 376*C.* 118*ft.* 1187*in.*, 489*C.* 76*ft.* 860*in.*, 776*C.* 114*ft.* 1560*in.*, and 678*C.* 112*ft.* 1600*in.* *Ans.* 2322*C.* 39*ft.* 23*in.*

EXPLANATIONS.

C.	ft.	in.
376	118	1187
489	76	860
776	114	1560
678	112	1600
2322	39	23

Begin, as before, at the right hand denomination, and add the inches; divide the amount by 1728, because seventeen hundred and twenty-eight solid inches are equal to one solid foot; set down the remainder, and carry the quotient to the column of feet. Add the feet; divide the amount by 128, because one hundred and twenty-eight solid feet are equal to one cord of wood; set down the remainder, and carry the quotient to the column of cords. Add the cords, and set down the whole amount, as in Simple Addition, because cord is the highest denomination in solid or cubick measure.

(2.)	(3.)	(4.)
T. ft.	C. ft.	ft. in.
49 27	10 114	24 1426
12 6	9 120	96 866
4 33	18 92	32 1446
39 26	7 123	100 1572
41 18	6 83	39 933

CLOTH MEASURE.

1. Add together 86*yd.* 1*qr.* 2*na.*, 55*yd.* 2*qr.* 3*na.*, 46*yd.* 1*qr.* 2*na.*, and 63*yd.* 3*qr.* 1*na.* *Ans.* 252*yd.* 1*qr.*

EXPLANATIONS.

yd.	*qr.*	*na.*
86	1	2
55	2	3
46	1	2
63	3	1
252	1	0

Begin, as before, at the right hand denomination, and add the nails; divide the amount by 4, because four nails are equal to one quarter; set down the remainder, and carry the quotient to the column of quarters. Add the quarters; divide the amount by 4, because four quarters are equal to one yard; set down the remainder, and carry the quotient to the column of yards. Add the yards, and set down the whole amount, as in Simple Addition, because yard is the highest denomination in cloth measure.

(2.)	(3.)	(4.)
E. Fl. qr. na.	*E. E. qr. na.*	*E. Fr. qr. na.*
45 1 2	48 4 2	63 5 2
63 2 1	86 3 1	44 3 3
89 1 3	14 1 3	36 2 2
83 2 1	9 2 2	14 4 1
27 1 2	36 4 1	25 1 3

CIRCULAR MOTION.

1. Add together 4*S*. 14° 18′ 10″, 5*S*. 19° 46′ 12″, 1*S*. 6° 10′ 17″, and 4*S*. 28° 17′ 14″. *Ans*. 16*S*. 8° 31′ 53″.

EXPLANATIONS.

S.	°	′	″
4	14	18	10
5	19	46	12
1	6	10	17
4	28	17	14
16	8	31	53

Begin, as before, at the right hand column, and add the seconds; divide the amount by 60, because sixty seconds are equal to one minute; set down the remainder, but in this example the whole amount, 53, is not equal to one minute, therefore, you must set down the 53, and carry nothing to the column of minutes. Add the minutes; divide the amount by 60, because sixty minutes are equal to one degree; set down the remainder, and carry the quotient to the column of degrees. Add the degrees; divide the amount by 30, because thirty degrees are equal to one sign; set down the remainder, and carry the quotient to the column of signs. Add the signs, and set down the whole amount, as in Simple Addition, because sign is the highest denomination in circular motion.

(2.)

S.	°	′	″
4	11	6	10
8	12	55	11
9	4	10	49
1	15	49	50
3	00	10	40

(3.)

S.	°	′	″
3	9	6	36
11	29	59	59
6	8	48	44
8	15	55	12
12	10	49	26

(4.)

°	′	″
19	50	49
10	3	57
16	54	39
9	36	23
17	44	50

PAPER.

1. Add together 19*ba.* 4*bun.* 1*r.* 18*q.* 16*s.*, 24*ba.* 3*bun.* 14*q.* 12*s.*, 36*ba.* 2*bun.* 1*r.* 17*q.* 19*s.*, and 48*ba.* 1*bun.* 1*r.* 12*q.* 22*s.* *Ans.* 129*ba.* 3*bun.* 0*r.* 3*q.* 21*s.*

EXPLANATIONS.

ba.	*bun.*	*r.*	*q.*	*s.*
19	4	1	18	16
24	3	0	14	12
36	2	1	17	19
48	1	1	12	22
129	3	0	3	21

Begin, as before, at the right hand column, and add the sheets; divide the amount by 24, because twenty-four sheets are equal to one quire; set down the remainder, and carry the quotient to the column of quires. Add the quires; divide the amount by 20, because twenty quires are equal to one ream; set down the remainder, and carry the quotient to the column of reams. Add the reams; divide the amount by 2, because two reams are equal to one bundle; set down the remainder, but in this example there is no remainder, therefore, you must set down a cipher, and carry the quotient to the column of bundles. Add the bundles; divide the amount by 5, because five bundles are equal to one bale; set down the remainder, and carry the quotient to the bales. Add the bales, and set down the whole amount, because bale is the highest denomination in paper.

(2.)

ba.	*bun.*	*r.*	*q.*	*s.*
29	1	1	13	16
38	4	0	16	19
14	2	1	15	21
56	3	0	14	17
17	2	1	13	23

(3.)

ba.	*bun.*	*r.*	*q.*	*s.*
14	1	0	17	23
93	2	1	19	14
36	1	1	12	17
41	2	1	13	21
29	4	0	16	14

(4.)

bun.	*r.*	*q.*
118	1	14
124	0	13
136	1	12
114	0	19
225	1	18

EXAMPLES

For Practical Exercise.

1. A gentleman paid $325,37,5 for a coach, $275,37,5 for a span of fine horses, and $75,25 for a set of harness; how much did he pay for all? *Ans.* $676.

2. James lent William at one time $15,25, at another $36,75, at another $75, and at another $25,37,5; how much did he lend him in all? *Ans.* $152,37,5.

3. A merchant deposited in the bank £125 6*s.* 7*d.* 2*qr.* at one time, £275 13*s.* 4*d.* 2*qr.* at another, and £5635 at another time; how much did he deposite in all? *Ans.* £6036.

4. George is 12*yr.* 3*mo.* 3*w.* 8*d.* old, William is 14*yr.* 7*mo.* 1*w.* 20*d.* old, Stephen is 9*yr.* 1*mo.* old, and Peter is 8*yr.* old; what is the sum of all their ages? *Ans.* 44*yr.*

5. A merchant bought 2*cwt.* 3*qr.* 14*lb.* of tea, 1*cwt.* 2*qr.* 23*lb.* 8*oz.* 8*dr.* of coffee, 5*cwt.* 14*lb.* of sugar, and 1*qr.* 4*lb.* 7*oz.* 8*dr.* of rice; what was the weight of the whole? *Ans.* 10*cwt.*

6. A physician bought drugs of an apothecary at one time, weighing 25℔ 6℥ 2ʒ 1℈ 18*gr.*, at another 4℔ 5℥ 5ʒ 1℈ 2*gr.*, and at another time 8℔; how much did he buy in all? *Ans.* 38℔.

7. A silversmith bought 3 ingots of silver; the first weighed 6*lb.* 4*oz.* 14*pwt.* 19*gr.*, the second 4*lb.* 8*oz.* 17*pwt.* 22*gr.*, and the third weighed 5*lb.* 6*oz.* 8*pwt.* 17*gr.*; what was the weight of the whole? *Ans.* 16*lb.* 8*oz.* 1*pwt.* 10*gr.*

8. A merchant bought of one farmer 17*bu.* 2*p.* 6*qt.* 1*pt.* of wheat, of another 12*bu.* 1*p.* 1*qt.* 1*pt.*, of another

25*bu.* 1*p.*, and of another 74*bu.* 3*p.*; how much did he buy in all? *Ans.* 130*bu.*

9. An innkeeper bought of a grocer 17*gal.* 3*qt.* 1*pt.* 3*gi.* of rum, 26*gal.* 1*qt.* of brandy, 10*gal.* 1*pt.* 1*gi.* of wine, and 29*gal.* 3*qt.* 1*pt.* of gin; how much did he buy in all? *Ans.* 84*gal.* 1*qt.*

10. A man travelled 25*m.* 5*fur.* 17*rd.* in one day, 36*m.* 2*fur.* 23*rd.* in another, and 38*m.* in another day; how far did he travel in all? *Ans.* 100*m.*

11. A farmer bought 4 farms; the first contained 115*a.* 1*r.* 10*po.*, the second 160*a.* 2*r.* 30*po.*, the third 210*a.* 3*r.* 20*po.*, and the fourth contained 215*a.* 20*po.*; how many acres did he buy in all? *Ans.* 702*a.*

12. A man brought 4 loads of wood to market; the first contained 1*C.* 63*ft.* 864*in.*, the second 2*C.* 32*ft.* 864*in.*, the third 1*C.* 34*ft.* 724*in.*, and the fourth contained 1*C.* 93*ft.* 1004*in.*; how many cords did he bring in all? *Ans.* 6*C.* 96*ft.*

13. A merchant bought 5 pieces of cloth; the first contained 36*yd.* 3*qr.* 1*na.*, the second 24*yd.* 2*qr.* 3*na.*, the third 23*yd.* 3*qr.* 1*na.*, the fourth 21*yd.* 1*qr.* 3*na.*, and the fifth contained 19*yd.* 1*qr.*; how many yards did he buy in all? *Ans.* 126*yd.*

14. A papermaker sold to a printer at one time 24*bun.* 1*r.* 16*q.*, at another 36*bun.* 14*q.*, at another 39*bun.* 1*r.* 10*q.*, and at another time 41*bun.* 1*r.*; how much did he sell him in all? *Ans.* 142*bun.* 1*r.*

NOTE.—TO TEACHERS. The learner should be questioned as often as once in each day respecting the nature and principles of the rules, and should not be permitted to commence a new sum, or engage in a new rule, until he is fully and thoroughly acquainted with the principles of the rule in which he has been working.

COMPOUND SUBSTRACTION.

Q. What is COMPOUND SUBSTRACTION?

A. Compound Substraction teaches to take a small number, or quantity, from a greater, of different denominations.

EXAMPLES

For Mental Exercise.

1. If you have twelve cents and five mills, and pay nine cents for an inkstand; how many cents will you have left?

2. James bought a penknife for twelve cents and five mills, and sold it for eighteen cents and five mills; how many cents did he make by the sale?

3. William paid twelve dollars and seventy-five cents for a coat, and four dollars and twenty-five cents for a hat; how much did he pay for his coat more than for his hat?

4. George has four dollars and thirty-seven cents and five mills, and Thomas has nine dollars and thirty-seven cents and five mills; how many dollars has Thomas more than George?

5. Rufus gave two shillings and six pence for a book, and one shilling and three pence for a slate; how much did he pay for the book more than for the slate?

6. Jane went to the store with six shillings. She paid four shillings for calico, and nine pence for riband; how much had she left?

7. Peter bought a watch for five pounds, six shillings, and six pence, and sold it for eight pounds, twelve shillings; how much did he make by the sale?

8. James learned his lesson in four hours and forty-five minutes, and William learned his in three hours and fifteen minutes; how much sooner did William learn his lesson than James did his?

9. One man walked from New York to Philadelphia in two days and six hours, and another man in three days and twelve hours; how much sooner did one perform the journey than the other?

10. James bought one pound and four ounces of raisins, and gave William eight ounces; how many ounces had he left?

11. A merchant bought three hundred-weight of sugar, and sold one hundred-weight and three quarters of it; how much had he left?

12. James bought four quarts and one pint of chestnuts, and William bought six quarts; how much had William more than James?

13. A merchant bought five bushels and three pecks of walnuts, and sold four bushels and one peck of them; how many had he left?

14. A man had one gallon and three quarts of wine in a bottle, and three quarts and one pint leaked out; how much had he left?

15. If you have a stick that is three feet and two inches long, and cut one foot and eight inches from it; how long will it then be?

16. A woman bought a piece of calico containing twelve yards, two quarters, and two nails, and gave five yards of it to a poor woman; what had she left?

Note.—To Teachers. The learner should be required to answer, mentally, the preceding questions, and various others of an equally simple nature, before he is required to use a slate.

RULE.

Q. How must the different numbers, or denominations, be placed in Compound Substraction?

A. They must be placed with like denominations, or numbers, directly under each other; as, mills under mills, cents under cents, dollars under dollars, ounces under ounces, pounds under pounds, quarts under quarts, &c.

Q. With which denomination must you begin to substract?

A. At the right hand or lowest denomination, as in Simple Substraction.

Q. Why do you begin to substract at the right hand denomination?

A. Because the different denominations increase in quantity from the right hand to the left, as in Simple Substraction.

Q. How must each denomination be Substracted?

A. It must be substracted from the denomination above it, as in Simple Substraction.

Q. When the number, or denomination, in the subtrahend, or lower line, is less than the number, or denomination, in the minuend, or upper line, how must it be substracted?

A. Substract and find the difference between them, and place it under the number or denomination you substracted from.

Q. When the number, or denomination, in

the subtrahend, or lower line, is greater than the number, or denomination, in the minuend, or upper line, how must it be substracted?

A. As many units must be borrowed as will make one in the next higher denomination, and added to the number in the upper line, or minuend, and then the number in the lower line, or subtrahend, must be substracted from the amount, and the difference set down.

Q. What must you carry to the next higher denomination in the lower line, for what you borrowed and added to the denomination in the upper line?

A. One must be added, or carried, to the next higher denomination in the lower line.

Q. How must the last or highest denomination be substracted?

A. It must be substracted as in Simple Substraction.

EXPLANATIONS.

Compound Substraction is precisely the reverse of Compound Addition; and your knowledge of Compound Addition will be of great assistance to you in the operations of Compound Substraction. You have learned that Compound Addition is joining, or adding, several numbers, or quantities, of different denominations into one sum. Now you must learn that Compound Substraction is separating numbers of different denominations, or taking them one from the other. Thus, by Compound Addition you learned. that four cents and five mills, and five

cents and five mills make ten cents. By Compound Substraction you will learn, that if the four cents and five mills, or the five cents and five mills be taken from the ten, the other number will remain; and you will also learn, that the number which remains after you have taken the other from the ten, shows the difference between the ten and the number which you took from it. Thus, five cents and five mills taken from ten cents leaves four cents and five mills; and four cents and five mills shows the difference between five cents and five mills and ten cents.

EXAMPLES

For Theoretical Exercise on a Slate.

FEDERAL MONEY.

1. James had 1 dollar 37 cents and 5 mills, and paid 75 cents for a book; how many cents had he left? *Ans.* 62 cents and 5 mills.

EXPLANATIONS.

$	c.	m.
1,	37,	5
	75,	0
	62,	5

Federal money increases in a tenfold proportion, and is, therefore, very nearly allied to whole numbers; and, consequently, the rule which you used in substracting whole numbers may be used here. You will remember, that the first thing to be done is to place the different denominations directly under each other; as, dollars under dollars, cents under cents, mills under mills. You must place a comma between the dollars, cents, and mills, and then substract as in whole numbers. If your sum consists of dollars and mills only, or of

dollars and cents only, you must place a cipher in the vacant place, as in the present example. Beginning at the right hand column of the subtrahend, the column of mills, you must say, cipher from 5 leaves 5; set down the 5 under the cipher, the column of mills, and as there is nothing to be carried to the first column of cents, you must begin anew. Thus, 5, in the first column of cents, taken from 7 leaves 2; set down the 2 under the 5, the first column of cents, and as there is nothing to be carried to the second column of cents, you must begin anew. Thus, 7, in the second column of cents, being larger than the 3, in the upper line, can not be taken from it, therefore, you must add ten to the 3 which makes it thirteen, and say, 7 from 13 leaves 6; set down the 6 under the 7, the second column of cents; and proceeding, you must say, one, borrowed, added to the next column, the place of dollars, from 1 leaves nothing, and as this is on the left hand, you need not set down the cipher. Thus, you will perceive, that he had 62 cents and 5 mills left after he had purchased the book.

PROOF.

The methods of proof are the same in Compound Substraction as in Simple Substraction.

2. William had 25 dollars, 37 cents, and 5 mills, and paid 18 dollars, 25 cents, and 5 mills for a watch; how much had he left? *Ans.* 7 dollars, 12 cents.

EXPLANATIONS.

$	c.	m.
25,	37,	5
18,	25,	5
7,	12,	0

You must begin, as before, with the right hand column, and say, 5 from 5 leaves nothing; set down a cipher under the column of mills, and proceed to the first column of cents. Thus, 5 from 7 leaves 2; set down the 2 under the first column of cents, and proceed to the second column of cents. Thus, 2 from 3 leaves 1; set down the 1 under the second column of cents, and proceed to the first column of dollars. Thus, 8 can not be taken from 5, therefore, you must add ten to the 5, which makes it 15, and say, 8 from 15 leaves 7; set down the 7 under the 8, and, proceeding, you must say, one borrowed, added to the 1 in the next column, makes 2, and 2 from 2 leaves nothing, and as this is on the left hand, you need not set down the cipher. Thus, you will perceive, that he had 7 dollars, 12 cents left after he had purchased the watch.

By paying particular attention to the use of the comma, in separating the dollars, cents, and mills, you will be able to work any sum in Substraction of federal money; for, indeed, the proper placing of the comma is the only operation which distinguishes this from Simple Substraction. I shall, therefore, give you a few examples for farther exercise, and pass to the next part of the subject.

(3.)	(4.)	(5.)
$ c. m.	$ c. m.	$ c. m.
9, 58, 4	37, 56, 1	736, 36, 1
8, 46, 3	12, 81, 9	481, 61, 4
1, 12, 1		

(6.)	(7.)	(8.)
\$ c. m.	\$ c. m.	\$ c. m.
5, 60, 3	98, 34, 5	964, 55, 2
2, 82, 1	12, 76, 5	813, 36, 4

STERLING OR ENGLISH MONEY.

1. A man bought a barrel of pork for £8 7*s.* 8*d.*, and a firkin of butter for £6 9*s.* 7*d.* 2*qr.*; how much more did he pay for the barrel of pork than for the firkin of butter? *Ans.* £1 18*s.* 0*d.* 2*qr.*

EXPLANATIONS.

£	s.	d.	qr.
8	7	8	0
6	9	7	2
1	18	0	2

As before, you have first to place the same denominations directly under each other; as, farthings under farthings, pence under pence, &c. Beginning with the lowest denomination, the farthings, you must say, 2 can not be taken from a cipher, therefore, you must borrow 4 and add to the upper line, the cipher, because four farthings are equal to one penny, the next higher denomination, and say, 2 from 4 leaves 2; set down the 2 under the column of farthings, and add, or carry, one to the column of pence, for the four which you borrowed and added to the cipher in the upper line. Thus, one, which you borrowed, added to the 7, in the column of pence, makes 8, and 8 from 8 leaves nothing; set down a cipher under the 7, the column of pence, and as there is nothing to be carried to the column of shillings, you must begin anew. Thus, 9 can not be taken from 7, therefore, you must borrow twenty, and add to the upper line, the 7, because

twenty shillings are equal to one pound, the next higher denomination, and say, 9 from 27 leaves 18; set down the 18 under the column of shillings, and add, or carry, one to the column of pounds, for the twenty which you borrowed and added to the 7 in the upper line. Thus, one, which you borrowed, added to the 6 makes 7, and 7 from 8 leaves 1; set down the 1 under the 6, the column of pounds, and then the work is done.

As sterling money is but very little used in this country, I shall merely give you a few sums for exercise.

(2.)	(3.)	(4.)
£ s. d. qr.	£ s. d. qr	£ s. d. qr.
98 14 4 1	963 17 7 2	223 9 10 3
36 10 6 3	142 8 8 1	189 12 8 0

TIME.

1. Substract 19*yr.* 14*d.* 13*h.* 55*min.* 56*sec.* from 28*yr.* 123*d.* 14*h.* 31*min.* 15*sec.* *Ans.* 9*yr.* 109*d.* 0*h.* 35*min.* 19*sec.*

EXPLANATIONS.

yr.	d.	h.	min.	sec.
28	123	14	31	15
19	14	13	55	56
9	109	0	35	19

In this example, you must begin, as before, with the lowest denomination, at the right hand, the seconds. You can not take 56 from 15, therefore, you must borrow sixty, and add to the 15, which will make it 75, because sixty seconds are equal to one minute, the next higher denomination,

and say, 56 from 75 leaves 19; set down the 19 under the 56, the column of seconds, and add, or carry, one to the column of minutes, for the sixty which you borrowed and added to the 15 in the upper line. Thus, one, which you borrowed, added to the 55, in the column of minutes, makes 56, and 56 can not be taken from 31, therefore, you must borrow sixty, and add to the 31, which will make it 91, because sixty minutes are equal to one hour, the next higher denomination, and say, 56 from 91 leaves 35; set down the 35 under the 55, the column of minutes, and add, or carry, one to the column of hours, for the sixty which you borrowed and added to the 31 in the upper line. Thus, one, which you borrowed, added to the 13, in the column of hours, makes 14, and 14 from 14 leaves nothing; set down a cipher under the 13, the column of hours, and as there is nothing to be carried to the column of days, you must begin anew. Thus, 14 from 123 leaves 109; set down the 109 under the column of days, and as there is nothing to be carried to the column of years, you must begin anew. Thus, as year is the highest denomination in time, you must substract the years as in Simple Substraction, and then the work is done.

Whenever the number of any denomination, which makes two or more columns, is smaller in the lower line than in the upper, as in the case of days and hours, in the present example, you must substract the different figures as in Simple Substraction, and set down the whole amount without any regard to the higher denominations. Thus, in the present example, 14 days are taken from 123. You must say 4 from 13 leaves 9; set down the 9, and carry one to the 1, and sav 2 from 2 leaves nothing; set

down a cipher, and as there is nothing to be carried, you must say, 1 is one, which you must set down at the left of the cipher, which will make it 109, the difference between 14 days and 123 days.

You should continually keep in mind, that the principles in Compound Arithmetick are the same as in Simple Arithmetick; the only difference is, that it takes different numbers in Compound Arithmetick to make one or a unit in the next higher; as, in seconds, it takes sixty to make a unit, or one in minutes; and, in minutes, it takes sixty to make a unit, or one in hours: in shillings it takes twenty to make a unit, or one in pounds: in quarts it takes eight to make a unit, or one in pecks; and, in pecks, it takes four to make a unit, or one in bushels, &c.

If you will pay strict attention to this difference, you will find no difficulty in making progress in Compound Arithmetick.

(2.)

yr.	*mo.*	*w.*	*d.*	*h.*	*min.*	*sec.*
96	3	1	5	21	13	29
12	10	2	4	19	12	42

(3.)

yr.	*d.*	*h.*	*min.*	*sec.*
92	124	13	51	52
24	63	15	44	46

AVOIRDUPOIS WEIGHT.

1. A merchant bought 4*T.* 19*cwt.* 3*qr.* 19*lb.* 9*oz.* 14*dr.* of cheese, and sold 3*T.* 14*cwt.* 1*qr.* 22*lb.* 11*oz.* 11*dr.* of it; how much had he left? *Ans.* 1*T.* 5*cwt.* 1*qr.* 24*lb.* 14*oz.* 3*dr.*

EXPLANATIONS.

T.	cwt.	qr.	lb.	oz.	dr.
4	19	3	19	9	14
3	14	1	22	11	11
1	5	1	24	14	3

Begin, as before, at the right hand, or lowest denomination, the drachms; substract the 11 from the 14, and set down the difference, which is 3. Proceeding to the ounces, you can not take 11 from 9, therefore, you must borrow sixteen and add to the 9, which will make it 25, because sixteen ounces are equal to one pound, the next higher denomination, and say, 11 from 25 leaves 14; set down the 14 under the 11, the column of ounces, and add, or carry, one to the column of pounds, for the sixteen which you borrowed and added to the 9 in the upper line. Thus, one, which you borrowed, added to the 22, in the column of pounds, makes 23, and 23 can not be taken from 19, therefore, you must borrow twenty-eight and add to the 19, which will make it 47, because twenty-eight pounds are equal to one quarter, the next higher denomination, and say, 23 from 47 leaves 24; set down the 24 under the 22, the column of pounds, and add, or carry, one to the column of quarters, for the twenty-eight which you borrowed and added to the 19 in the upper line. Thus, one, which you borrowed, added to the 1, in the column of quarters, makes 2, and 2 from 3 leaves 1; set down the 1 under the 1, the column of quarters. Proceeding to the 14, in the column of hundred-weight, as there is nothing to be carried to that, you must say, 14 from 19 leaves 5; set down the 5 under the 14, in the column of hundred-weight. Proceeding to the 3, in the column of tuns, as there is nothing to be carried to that, you must say, 3 from 4 leaves 1; set

down the 1 under the 3, in the column of tuns, and then the work is done.

(2.)

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
96	19	3	27	15	11
77	6	2	3	2	13

(3.)

cwt.	*qr.*	*lb.*	*oz.*	*dr*
17	2	2	3	8
10	1	19	13	12

APOTHECARIES WEIGHT.

1. Substract 24℔ 4℥ 4ʒ 1℈ 13*gr.* from 27℔ 10℥ 1ʒ 1℈ 14*gr.* *Ans.* 3℔ 5℥ 5ʒ 0℈ 1*gr*

EXPLANATIONS.

℔	℥	ʒ	℈	*gr.*
27	10	1	1	14
24	4	4	1	13
3	5	5	0	1

Begin, as before, at the right hand denomination; substract the 13 from the 14, in the column of grains, and set down the difference, which is 1. Substract the 1 from the 1, in the column of scruples, and set down a cipher as there is no difference. Proceeding to the drachms, you can not take 4 from 1, therefore, you must borrow eight and add to the 1, which will make it 9, because eight drachms are equal to one ounce, the next higher denomination, and say, 4 from 9 leaves 5; set down the 5 under the 4, the column of drachms and add, or carry, one to the column of ounces, for the eight which you borrowed and added to the 1 in the upper line. Thus, one, which you borrowed, added to the 4, in the column of ounces, makes 5, and 5 from 10 leaves 5; set down the 5 under the column of ounces. Proceeding to the 4 in the column of pounds, you must say, 4 from 7 leaves 3; set down the 3, and then the work is done.

(2.)

℔	℥	ʒ	℈	gr.
57	6	3	2	16
42	5	1	0	19

(3.)

℔	℥	ʒ	℈	gr.
12	9	2	1	6
9	0	0	2	10

TROY WEIGHT.

1. Substract 27*lb.* 10*oz.* 17*pwt.* 18*gr.* from 117*lb.* 9*oz.* 18*pwt.* 14*gr.* *Ans.* 89*lb.* 11*oz.* 0*pwt.* 20*gr.*

EXPLANATIONS.

lb.	*oz.*	*pwt.*	*gr.*
117	9	18	14
27	10	17	18
89	11	0	20

Begin, as before, at the right hand denomination to substract. When the number in the lower line of grains is larger than the upper, borrow twenty-four and add to the number in the upper line, because twenty-four grains are equal to one penny-weight, the next higher denomination; substract the number in the lower line from the amount, and carry one to the column of penny-weights, for the twenty-four which you borrowed. When the number in the lower line of penny-weights is larger than the upper, borrow twenty, and add to the number in the upper line, because twenty penny-weights are equal to one ounce, the next higher denomination; substract the number in the lower line from the amount, and carry one to the column of ounces for the twenty which you borrowed. When the number in the lower line of ounces is larger than the upper, borrow twelve, and add to the number in the upper line, because twelve ounces are equal to one pound, the next higher denomination; substract the num-

ber in the lower line from the amount, and carry one to the column of pounds, for the twelve which you borrowed. Substract the pounds as in Simple Substraction, because pound is the highest denomination in troy weight.

(2.)

lb.	*oz.*	*pw.*	*gr.*
12	9	19	21
2	8	10	22

(3.)

lb.	*oz.*	*pwt.*	*gr.*
16	11	14	11
10	4	15	19

DRY MEASURE.

1. A farmer had 90*bu.* 3*p.* 4*qt.* 1*pt.* of wheat, and sold 31*bu.* 2*p.* 6*qt.* 1*pt.* of it; how much had he left? *Ans.* 59*bu.* 0*p.* 6*qt.* 0*pt.*

EXPLANATIONS.

bu.	*p.*	*qt.*	*pt.*
90	3	4	1
31	2	6	1
59	0	6	0

Begin, as before, at the right hand denomination to substract. When the lower line of pints is larger than the upper, borrow two and add to the upper line, because two pints are equal to one quart; substract the lower line from the amount, and carry one to the column of quarts. When the lower line of quarts is larger than the upper, borrow eight, and add to the upper line, because eight quarts are equal to one peck; substract the lower line from the amount, and carry one to the column of pecks. When the lower line of pecks is larger than the upper, borrow four and add to the upper line, because four pecks are equal to one bushel; substract the lower line from the amount,

and carry one to the column of bushels. Substract the bushels as in Simple Substraction, because bushel is the highest denomination in dry measure, or in the measure of wheat, &c., as in the present example

(2.)

bu.	*p.*	*qt.*	*pt.*
23	2	7	0
17	3	5	1

(3.)

ch.	*bu.*	*p.*	*qt.*	*pt.*
28	24	3	7	1
13	28	2	3	1

WINE MEASURE.

1. A merchant had 11*T.* 1*p.* 1*hhd.* 54*gal.* 3*qt.* 1*pt.* 1*gi.* of wine, and sold 9*T.* 1*p.* 55*gal.* 2*qt.* 1*pt.* 1*gi.* of it; how much had he left? *Ans.* 2*T.* 0*p.* 0*hhd.* 62*gal.* 1*qt.* 0*pt.* 0*gi.*.

EXPLANATIONS.

T.	*p.*	*hhd.*	*gal.*	*qt.*	*pt.*	*gi.*
11	1	1	54	3	1	1
9	1	0	55	2	1	1
2	0	0	62	1	0	0

Begin, as before, at the right hand denomination to substract. When the lower line of gills is larger than the upper, borrow four and add to the upper line, because four gills are equal to one pint; substract the lower line from the amount, and carry one to the column of pints. When the lower line of pints is larger than the upper, borrow two and add to the upper line, because two pints are equal to one quart; substract the lower line from the amount, and carry one to the column of quarts. When the lower line of quarts is larger than the upper, borrow four and add to the upper line, because four quarts are equal

to one gallon; substract the amount, and carry one to the column of gallons. When the lower line of gallons is larger than the upper, borrow sixty-three, if the next higher denomination be hogsheads, as in the present example, and add to the upper line, because sixty-three gallons are equal to one hogshead; or, if the next higher denomination be barrels, borrow thirty-one and a half, because thirty-one and a half gallons are equal to one barrel; if tierces, borrow forty-two, because forty-two gallons are equal to one tierce; and if puncheons, borrow eighty-four, because eighty-four gallons are equal to one puncheon; substract the amount, and carry one to the next higher denomination, whether hogsheads, tierces, barrels, or puncheons. When the lower line of hogsheads is larger than the upper, borrow two and add to the upper line, because two hogsheads are equal to one pipe; substract the lower line from the amount, and carry one to the column of pipes. When the lower line of pipes is larger than the upper, borrow two and add to the upper line, because two pipes are equal to one tun; substract the lower line from the amount, and carry one to the column of tuns. Substract the tuns as in Simple Substraction, because tun is the highest denomination in wine measure.

By reading over the EXPLANATIONS with care and attention, you will soon be able to perform every operation in Compound Substraction.

(2.)

bar.	*gal.*	*qt.*	*pt.*	*gi.*
27	25	1	0	3
16	17	3	1	2

(3.)

tie.	*gal.*	*qt.*
36	24	3
28	31	2

(4.)

pun.	*gal.*	*qt.*
47	77	2
34	81	1

LONG MEASURE.

1 Substract 10*de.* 10*m.* 5*fur.* 24*rd.* 4*yd.* 2*ft.* 9*in.* 2*bc.* from 56*de.* 54*m.* 7*fur.* 35*rd.* 5*yd.* 2*ft.* 10*in.* 0*bc.* *Ans.* 46*de.* 44*m.* 2*fur.* 11*rd.* 1*yd.* 0*ft.* 0*in.* 1*bc.*

EXPLANATIONS.

de.	m.	fur.	rd.	yd.	ft.	in.	bc.
56	54	7	35	5	2	10	0
10	10	5	24	4	2	9	2
46	44	2	11	1	0	0	1

Begin, as before, at the right hand denomination to substract. When the lower line of barley-corns is larger than the upper, borrow three and add to the upper line, because three barley-corns are equal to one inch; substract the lower line from the amount, and carry one to the column of inches. When the lower line of inches is larger than the upper, borrow twelve and add to the upper line, because twelve inches are equal to one foot; substract the lower line from the amount, and carry one to the column of feet. When the lower line of feet is larger than the upper, borrow three and add to the upper line, because three feet are equal to one yard; substract the lower line from the amount, and carry one to the column of yards. When the lower line of yards is larger than the upper, borrow five and a half and add to the upper line, because five and a half yards are equal to one rod; substract the lower line from the amount, and carry one to the column of rods. When the lower line of rods is larger than the upper, borrow forty and add to the upper line, because forty rods are equal to one furlong; substract the lower line from the amount, and carry one to the column of furlongs. When

the lower line of furlongs is larger than the upper, borrow eight and add to the upper line, because eight furlongs are equal to one mile; substract the lower line from the amount, and carry one to the column of miles. When the lower line of miles is larger than the upper, borrow sixty and add to the upper line, because sixty geographick miles are equal to one degree; substract the lower line from the amount, and carry one to the column of degrees. Substract the degrees as in Simple Substraction, because degree is the highest denomination in long measure.

(2.)

yd.	*ft.*	*in.*	*bc.*
36	0	8	1
29	1	7	2

(3.)

m.	*fur.*	*rd.*	*yd.*	*ft.*	*in.*	*bc.*
678	3	29	4	1	10	2
136	5	8	2	2	9	1

LAND OR SQUARE MEASURE.

1. Substract 219*m.* 150*a.* 3*r.* 30*po.* 5*yd.* 5*ft.* 14*in.* from 431*m.* 320*a.* 2*r.* 38*po.* 9*yd.* 6*ft.* 140*in.* *Ans.* 212*m.* 169*a.* 3*r.* 8*po.* 4*yd.* 1*ft.* 126*in.*

EXPLANATIONS

m.	*a.*	*r.*	*po.*	*yd.*	*ft.*	*in.*
431	320	2	38	9	6	140
219	150	3	30	5	5	14
212	169	3	8	4	1	126

Begin, as before, at the right hand denomination to substract. When the lower line of inches is larger than the upper, borrow one hundred and forty-four and add to the upper line, because one hundred and forty-four square inches are equal to one square foot; substract the lower line from the

amount, and carry one to the column of feet. When the lower line of feet is larger than the upper, borrow nine and add to the upper line, because nine square feet are equal to one square yard; substract the lower line from the amount, and carry one to the column of yards. When the lower line of yards is larger than the upper, borrow thirty and a quarter and add to the upper line, because thirty and a quarter square yards are equal to one square pole or rod; substract the lower line from the amount, and carry one to the column of poles or rods. When the lower line of poles or rods is larger than the upper, borrow forty and add to the upper line, because forty square poles or rods are equal to one square rood; substract the lower line from the amount, and carry one to the column of roods. When the lower line of roods is larger than the upper, borrow four and add to the upper line, because four square roods are equal to one square acre; substract the lower line from the amount, and carry one to the column of acres. When the lower line of acres is larger than the upper, borrow six hundred and forty and add to the upper line, because six hundred and forty square acres are equal to one square mile; substract the lower line from the amount, and carry one to the column of miles. Substract the miles as in Simple Substraction, because mile is the highest denomination in land or square measure.

(2.)

a.	*r.*	*po.*	*yd.*	*ft.*	*in.*
245	3	27	29	3	143
136	3	14	30	7	137

(3.)

a.	*r.*	*po.*
267	2	38
195	1	24

SOLID OR CUBICK MEASURE.

1. Substract 76*C.* 112*ft.* 860*in.* from 89*C.* 109*ft.* 1187*in.* *Ans.* 12*C.* 125*ft.* 327*in.*

EXPLANATIONS.

C.	*ft.*	*in.*
89	109	1187
76	112	860
12	125	327

Begin, as before, at the right hand denomination to substract. When the lower line of inches is larger than the upper, borrow seventeen hundred and twenty-eight and add to the upper line, because seventeen hundred and twenty-eight solid inches are equal to one solid foot; substract the lower line from the amount, and carry one to the column of feet. When the lower line of feet is larger than the upper, borrow one hundred and twenty-eight and add to the upper line, if the next higher denomination be cords, if it be round timber borrow forty, if it be hewn timber borrow fifty, and add to the upper line, because one hundred and twenty-eight solid feet are equal to one cord; forty feet of round timber, and fifty feet of hewn timber one tun; substract the lower line from the amount, and carry one to the next column, whether cords or tuns. Substract the cords as in Simple Substraction, because cord is the highest denomination in solid or cubick measure.

(2.)		(3.)		(4.)	
T.	*ft.*	*C.*	*ft.*	*ft.*	*in.*
49	27	10	114	96	1426
12	6	9	120	24	866

CLOTH MEASURE.

1. A merchant bought a piece of cloth containing 86*yd.* 1*qr.* 2*na.*, and sold 55*yd.* 2*qr.* 3*na.* of it; how much had he left? *Ans.* 30*yd.* 2*qr.* 3*na.*

EXPLANATIONS.

yd.	*qr.*	*na.*
86	1	2
55	2	3
30	2	3

Begin, as before, at the right hand denomination to substract. When the lower line of nails is larger than the upper, borrow four and add to the upper line, because four nails are equal to one quarter of a yard; substract the lower line from the amount, and carry one to the column of quarters. When the lower line of quarters is larger than the upper, borrow four and add to the upper line, because four quarters are equal to one yard; substract the lower line and carry one to the column of yards. Substract the yards as in Simple Substraction, because yard is the highest denomination in cloth measure.

(2.)			(3.)			(4.)		
E.Fl.	*qr.*	*na.*	*E.E.*	*qr.*	*na.*	*E.Fr.*	*qr.*	*na.*
63	2	1	86	4	1	63	5	2
45	1	2	48	3	2	44	3	3

CIRCULAR MOTION.

1. Substract 4*S.* 14° 18′ 10″ from 5*S.* 19° 46′ 9″. *Ans.* 1*S.* 5° 27′ 59″.

EXPLANATIONS.

Begin, as before, at the right hand denomination to substract. When the lower line of seconds is larger than the upper, borrow sixty and add to the upper line, because sixty seconds are equal to one minute; substract the lower line from the amount, and carry one to the column of minutes. When the lower line of minutes is larger than the upper, borrow sixty and add to the upper line, because sixty minutes are equal to one degree; substract the lower line from the amount, and carry one to the column of degrees. When the lower line of degrees is larger than the upper, borrow thirty and add to the upper line, because thirty degrees are equal to one sign, and carry one to the column of signs. Substract the signs as in Simple Substraction, because sign is the highest denomination in circular motion.

S.	°	′	″
5	19	46	9
4	14	18	10
1	5	27	59

(2.)

S.	°	′	″
8	12	55	11
4	11	56	9

(3.)

S.	°	′	″
11	29	59	59
3	9	6	36

(4.)

°	′	″
10	50	49
9	57	51

PAPER.

1. A papermaker had 24*ba.* 4*bun.* 1*r.* 18*q.* 16*s.*, and sold 19*ba.* 3*bun* 1*r.* 12*q.* 22*s.* to a printer; how much had he left? *Ans.* 5*ba.* 1*bun.* 0*r.* 5*q.* 18*s.*

EXPLANATIONS.

ba.	bun.	r.	q.	s.
24	4	1	18	16
19	3	1	12	22
5	1	0	5	18

Begin, as before, at the right hand denomination to substract. When the lower line of sheets is larger than the upper, borrow twenty-four and add to the upper line, because twenty-four sheets are equal to one quire; substract the lower line from the amount, and carry one to the column of quires. When the lower line of quires is larger than the upper, borrow twenty and add to the upper line, because twenty quires are equal to one ream; substract the lower line from the amount, and carry one to the column of reams. When the lower line of reams is larger than the upper, borrow two and add to the upper line, because two reams are equal to one bundle; substract the lower line from the amount, and carry one to the column of bundles. When the lower line of bundles is larger than the upper, borrow five and add to the upper line, because five bundles are equal to one bale; substract the lower line from the amount, and carry one to the column of bales. Substract the bales as in Simple Substraction, because bale is the highest denomination in paper.

(2.)

ba.	bun.	r.	q.	s.
38	1	1	16	19
29	0	1	13	16

(3.)

ba.	bun.	r.	q.	s.
93	1	1	19	14
36	2	0	15	23

(4.)

ba.	r.	q.
117	1	14
114	1	17

EXAMPLES

For Practical Exercise.

1. A gentleman paid \$325,37,5 for a coach, and \$275,25 for a span of fine horses; how much more did he pay for the coach than for the horses? *Ans.* \$50,12,5.

2. William lent James \$36,75; he has paid him \$15,42; how much remains unpaid? *Ans.* \$21,33.

3. A merchant deposited £275 13*s.* 4*d.*, and drew out £125 6*s.* 7*d.*; how much did he leave in the bank? *Ans.* £150 6*s.* 9*d.*

4. A merchant bought 2*cwt.* 3*qr.* 16*lb.* 8*oz.* of tea, and sold 2*cwt.* 3*qr.* 14*lb.* 4*oz.* of it; how much has he left? *Ans.* 2*lb.* 4*oz.*

5. A silversmith bought 6*lb.* 4*oz.* 14*pwt.* 19*gr.* of silver, and manufactured 4*lb.* 6*oz.* 19*pwt.* 16*gr.*, how much has he unmanufactured? *Ans.* 1*lb.* 9*oz.* 15*pwt.* 3*gr.*

6. A merchant bought 74*bu.* 3*p.* 4*qt.* of wheat, and sold 56*bu.* 2*p.* of it; how much had he left? *Ans.* 18*bu.* 1*p.* 4*qt.*

7. A grocer bought 5*hhd.* 17*gal.* 3*qt.* of brandy, and sold 3*hhd.* 57*gal.* 2*qt.* 1*pt.* of it; how much had he left? *Ans.* 1*hhd.* 23*gal.* 0*qt.* 1*pt.*

8. A man walked 25*m.* 5*fur.* 25*rd.* in one day, and 36*m.* 6*fur.* 27*rd.* in another; how much farther did he travel the second day than the first? *Ans.* 11*m.* 1*fur.* 2*rd.*

9. A farmer had 215*a.* 1*r.* 20*po.*, and gave 127*a.* 3*r.* 19*po.* to his son; how much had he left? *Ans.* 87*a.* 2*r.* 1*po.*

10. A man brought 127*C.* 66*ft.* of wood to market, and sold 93*C.* 94*ft.*; how much had he left? *Ans.* 33*C.* 100*ft.*

11. A merchant bought 575*yd.* 3*qr.* of cloth, and sold 434*yd.* 2*qr.* of it; how much had he left? *Ans.* 141*yd.* 1*qr.*

12. A paper maker had 179*ba.* 2*bun.* of paper, and sold to a printer 156*ba.* 1*bun.*; how much had he left? *Ans.* 23*ba.* 1*bun.*

COMPOUND MULTIPLICATION.

Q. What is Compound Multiplication?

A. Compound Multiplication teaches a short way of doing Compound Addition.

EXAMPLES

For Mental Exercise.

1. If you pay two cents and five mills for one orange; how many cents do you pay for four oranges?

2. If James paid fifteen cents for one penknife; how many cents did he pay for four penknives?

3. If one pair of shoes cost one dollar and twenty-five cents; what will eight pairs cost?

4. A man bought eight cows at twelve dollars and fifty cents each; how many dollars did he pay for the whole?

5. Jane bought eight yards of riband at four pe1 and two farthings a yard; how many shillings d she pay for the whole?

6. A lady bought six yards of calico at one shilling and six pence a yard; how many shillings did she pay for the whole?

7. If James can learn one lesson in one hour and fifteen minutes; how many hours will it require for him to learn four lessons?

8. A farmer brought eight fowls to market, each weighing seven pounds and four ounces; how many pounds did they all weigh?

9. Four boys gathered chestnuts, and each filled his basket, containing four quarts and one pint; how many pecks had they in the four baskets?

10. A farmer brought eight bags of wheat to market, each containing three bushels and four quarts; how many bushels were there in all?

11. A gentleman bought eight bottles of wine, each containing one quart and one pint; how many gallons were there in all?

12. A lady bought four pieces of calico, each containing six yards and two quarters; how many yards were there in all?

Note.—To Teachers. The learner should be required to answer, mentally, the preceding questions, and various others of an equally simple nature, before he is required to use a slate.

RULE.

Q. How must the different numbers, or quantities to be multiplied, be placed in Compound Multiplication?

A. They must be placed with the lowest denomination at the right hand, and the highest at the left, as in Compound Addition.

Q. Where must you begin to multiply in Compound Multiplication?

A. At the right hand, or lowest denomination, as in Simple Multiplication.

Q. Why do you begin to multiply at the right hand denomination?

A. Because the different denominations increase in quantity from the right hand to the left, as in Simple Multiplication.

Q. Where must the multiplier be placed?

A. It must be placed under the lowest, or right hand denomination of the multiplicand.

Q. How must you multiply each denomination, and what sum must you set down, and what must you carry to the next column in Compound Multiplication?

A. Each denomination must be multiplied separately, and the amount must be divided by the number that it takes of that denomination to make one in the next higher denomination, and the remainder, if there be any, must be set down, and the quotient must be carried to the next higher denomination, as in Compound Addition.

Q. How must the left hand, or highest denomination be multiplied?

A. It must be multiplied, and the whole amount set down, as in Simple Multiplication.

EXAMPLES

For Theoretical Exercise on a Slate.

FEDERAL MONEY.

1. If a man can earn 1 dollar 37 cents and 5 mills in one day; how much can he earn in six days? *Ans.* $8,25,0.

EXPLANATIONS.

$	c.	m.
1,	37,	5
		6
8,	25,	0

You will remember, that federal money increases in a tenfold proportion, and is, therefore, nearly allied to whole numbers; consequently, the rule which you used in multiplying whole numbers may be used here; the only difference is the proper placing of the comma, in separating the dollars, cents, and mills. You must also remember, that the first thing to be done is to place the different denominations with the lowest at the right hand, as, mills, cents, and dollars; place a comma between the mills, cents, and dollars. Then place the multiplier under the lowest denomination, and multiply as in whole numbers. If your sum consists of dollars and mills only, or of dollars and cents only, you must place a cipher in the vacant place, as in Compound Addition. Beginning, you must say, 6 times 5 are thirty, that is, thirty mills; set down a cipher in the place of mills, and carry three to the right hand column of cents. Thus, 6 times 7 are forty-two, and the three carried make forty-five; set down 5 under the right hand column of cents, and carry four to the second column of cents. Thus, 6 times 3 are eighteen, and the four carried make twenty-two; set down

2 under the second, or left hand column of cents, and carry two to the column of dollars. Thus, 6 times 1 are six, and the two carried make eight; set down 8 under the column of dollars, and then the work is done.

Compound Multiplication is, as you have been told, a short way of doing Compound Addition. Thus, you see that $1,37,5, multiplied by 6, increases it to $8,25,0; that is, $1,37,5 six times repeated. You will readily see, that this joining together of the same amount, several times repeated, may be accomplished by Compound Addition; first, by writing down the figures of the multiplicand as many times as there are units in the multiplier, in columns, and then by adding them up. But the end is attained much more quickly, more pleasantly, and with less liability to errour, by Compound Multiplication. Thus, by Compound Addition, you must set down the $1,37,5 six times, and add them up.

$ *c. m.*
1,37,5
1,37,5
1,37,5
1,37,5
1,37,5
1,37,5
8,25,0

PROOF.

The methods of proof are the same in Compound Multiplication as in Simple Multiplication.

(2.)	(3.)	(4.)
$ *c. m.*	$ *c. m.*	$ *c. m.*
8,46,4	37,56,1	736,36,0
3	4	5

(5.)	(6.)	(7.)
\$ c. m.	\$ c. m.	\$ c. m.
9,58,3	12,81,0	481,61,4
6	7	8

(8.)	(9.)	(10.)
\$ c. m.	\$ c. m.	\$ c. m.
6,33,8	61,16,3	218,63,8
9	11	12

STERLING OR ENGLISH MONEY.

1. A man bought 5 firkins of butter for £6 9*s*. 7*d*. 2*qr* each; how much did he pay for the whole? *Ans*. £32 8*s*. 1*d*. 2*qr*.

EXPLANATIONS.

£	s.	d.	qr.
6	9	7	2
			5
32	8	1	2

As before directed, you must place the multiplier under the lowest denomination, and then multiply. Beginning, you must say, 5 times 2 are ten, that is, ten farthings; divide the amount by 4, because four farthings make one penny; set down the remainder, which is 2, and carry the quotient, which is two, to the column of pence. Thus, 5 times 7 are thirty-five, and two carried make thirty-seven, that is, thirty-seven pence; divide the amount by 12, because twelve pence make one shilling; set down the remainder, which is 1, and carry the quotient, which is three, to the column of shillings. Thus, 5 times 9 are forty-five, and three carried make forty-eight, that is, forty-eight shillings;

divide the amount by 20, because twenty shillings make one pound; set down the remainder, which is 8, and carry the quotient, which is two, to the column of pounds. Thus, 5 times 6 are thirty, and two carried make 32; and as pound is the highest denomination, you must set down the whole amount, as in Simple Multiplication.

As before stated, the principles of Compound Multiplication do not materially differ from those of Simple Multiplication. Thus, in Simple Multiplication you carry one for every ten; but in Compound Multiplication you carry by the number that it takes to make, or equal one in the next higher denomination; this is all the difference.

(2.)	(3.)	(4.)
£ *s.* *d.* *qr.*	£ *s.* *d.* *qr.*	£ *s.* *d.* *qr.*
36 10 6 3	142 8 7 2	189 12 8 0
8	9	12

TIME.

1. Multiply 28*yr.* 123*d.* 14*h.* 31*min.* 15*sec.* by 4. *Ans.* 113*yr.* 129*d.* 10*h.* 5*min.* 0*sec.*

EXPLANATIONS.

yr.	*d.*	*h.*	*min.*	*sec.*
28	123	14	31	15
				4
113	129	10	5	0

Begin, as before, with the lowest denomination, at the right hand, to multiply. Multiply the seconds; divide the amount by 60, because sixty seconds make one minute; set down the remainder, but in this example there is no

remainder, therefore, you must set down a cipher, and carry the quotient, which is one, to the column of minutes. Multiply the minutes; divide the amount by 60, because sixty minutes make one hour; set down the remainder, which is 5, and carry the quotient, which is two, to the column of hours. Multiply the hours; divide the amount by 24, because twenty-four hours make one day; set down the remainder, which is 10, and carry the quotient, which is two, to the column of days. Multiply the days; divide the amount by 365, because three hundred and sixty-five days make one year; set down the remainder, which is 129, and carry the quotient, which is one, to the column of years. Multiply the years, and set down the whole amount, because year is the highest denomination in time.

(2.)

yr.	*mo.*	*w.*	*d.*	*h.*	*min.*	*sec.*
12	10	2	4	19	34	56
						9

(3.

yr.	*d.*	*h.*	*min.*	*sec.*
24	124	13	51	52
				11

AVOIRDUPOIS WEIGHT.

1. Multiply 13*T.* 3*cwt.* 2*qr.* 19*lb.* 9*oz.* 8*dr.* by 4.
Ans. 52*T.* 14*cwt.* 2*qr.* 22*lb.* 6*oz.* 0*dr.*

EXPLANATIONS.

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
13	3	2	19	9	8
					4
52	14	2	22	6	0

Begin, as before, with the lowest denomination, at the rignt hand, to multiply. Multiply the drachms; divide the amount by 16, because sixteen

drachms make one ounce; set down the remainder, but in this example there is no remainder, therefore, you must set down a cipher, and carry the quotient, which is two, to the column of ounces. Multiply the ounces; divide the amount by 16, because sixteen ounces make one pound; set down the remainder, which is 6, and carry the quotient, which is two, to the column of pounds. Multiply the pounds; divide the amount by 28, because twenty-eight pounds make one quarter; set down the remainder, which is 22, and carry the quotient, which is two, to the column of quarters. Multiply the quarters; divide the amount by 4, because four quarters make one hundred-weight; set down the remainder, which is 2, and carry the quotient, which is two, to the column of hundred-weight. Multiply the hundred-weight; divide the amount by 20, because twenty hundred-weight make one tun; set down the remainder, which is 14, and carry the quotient, but in this example there is no quotient, as the whole amount of hundred-weight, 14, does not make, or equal one tun, therefore, there is nothing to be carried to the column of tuns. Multiply the tuns, and set down the whole amount, because tun is the highest denomination in avoirdupois weight.

You must always remember, that when the amount of an inferiour, or lower denomination, does not amount to, or equal one of the next higher denomination, the whole must be set down, and that nothing is then to be carried to the next denomination.

By paying particular attention to the preceding EXPLANATIONS, you will be able to work any sum in Compound Multiplication; that is, any sum in which the multiplier does not exceed twelve.

(2.)					
T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
77	19	3	27	15	11
					7

(3.)				
cwt.	*qr.*	*lb.*	*oz.*	*dr.*
5	0	8	9	7
				12

APOTHECARIES WEIGHT.

(1.)				
℔	℥	ʒ	℈	*gr.*
27	10	1	2	18
				5

(2.)				
℔	℥	ʒ	℈	*gr.*
17	9	7	1	14
				9

TROY WEIGHT.

(1.)			
lb.	*oz.*	*pwt.*	*gr.*
12	9	19	21
			8

(2.)			
lb.	*oz.*	*pwt.*	*gr.*
96	7	16	22
			9

DRY MEASURE.

(1.)			
bu.	*p.*	*qt.*	*pt.*
22	2	7	1
			7

(2.)			
bu.	*p.*	*qt.*	*pt.*
90	0	6	0
			11

(3.)			
bu.	*p.*	*qt.*	*pt.*
11	1	5	1
			12

WINE MEASURE.

(1.)						
T.	*p.*	*hhd.*	*gal.*	*qt.*	*pt.*	*gi.*
13	1	1	54	3	1	3
						7

(2.)				
bar.	*gal.*	*qt.*	*pt.*	*gi.*
27	25	1	0	2
				9

LONG MEASURE.

(1.)

de.	m.	fur.	rd.	yd.	ft.	in.	bc.
10	10	5	24	4	2	9	2
							6

(2.)

yd.	ft.	in.	bc.
36	0	7	1
			5

LAND OR SQUARE MEASURE.

(1.)

m.	a.	r.	po.	yd.	ft.	in.
219	150	3	38	5	5	114
						8

(2.)

a.	r.	p.
195	1	24
		9

SOLID OR CUBICK MEASURE.

(1.)

C.	ft.	in.
376	118	1187
		4

(2.)

T.	ft.
49	27
	5

(3.)

C.	ft.
10	114
	6

CLOTH MEASURE.

(1.)

yd.	qr.	na.
86	1	2
		4

(2.)

E.Fl.	qr.	na.
45	1	2
		5

(3.)

E.E.	qr.	na.
48	4	2
		6

(4.)

E.Fr.	qr.	na.
63	5	2
		7

CIRCULAR MOTION.

S.	°	'	"
4	11	6	10
			7

S.	°	'	"
3	9	6	37
			6

°	'	"
19	50	49
		5

PAPER.

(1.)					(2.)				
ba.	*bun.*	*r.*	*q.*	*s.*	*ba.*	*bun.*	*r.*	*q.*	*s.*
24	4	1	18	16	38	1	1	16	19
				6					7

RULE.

Q. When the multiplier exceeds 12, and is a composite number, that is, when any two numbers in the multiplication table, being multiplied together, will produce a number exactly equal to the multiplier, how must you multiply?

A. Multiply first by one of those numbers or component parts; then multiply that product by the other number, and the last product will be the total product, as in Simple Multiplication.

EXAMPLES

For Theoretical Exercise on a Slate.

1. A farmer sold 21 bushels of wheat for \$1,37,5 a bushel; how many dollars did he receive for the whole? *Ans.* \$28,87,5.

EXPLANATIONS.

Here, in this example, 3 and 7 are the component parts of 21; for 3 times 7 are 21. You must first multiply by the 3, which will repeat the multiplicand three times; that is, it shows the price of three bushels; and you must multiply this product, $4,12,5 by 7, which will repeat it seven times; that is, it shows a number seven times as large as the first product of	$ c. m. 1,37,5 3 ——— 4,12,5 7 ——— 28,87,5

the $1,37,5 repeated three times; and, therefore, it shows the price of 21 bushels at $1,37,5 a bushel, as three times seven are twenty-one.

(2.)	(3.)	(4.)
$ c. m.	$ c. m.	$ c. m.
2,60,3	12,34,5	271,35,6
16	18	24

(5.)	(6.)	(7.)
£ s. d. qr.	£ s. d. qr.	£ s. d. qr.
61 7 0 3	32 11 6 1	823 8 4 2
27	28	32

(8.)	(9.)
yr. mo. w. d. h. min. sec.	yr. d. h. min. sec.
36 7 1 2 14 51 37	77 96 13 13 9
33	36

(10.)

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
19	17	1	24	9	7
					42

(11.

cwt.	*qr.*	*lb.*	*oz.*	*dr.*
19	2	23	15	9
				44

(12.)

℔	℥	ʒ	℈	*gr.*
71	3	7	1	12
				48

(13.)

℔	℥	ʒ	℈	*gr.*
18	11	6	2	12
				54

(14.)

lb.	*oz.*	*pwt.*	*gr.*
16	11	14	11
			56

(15.)

lb.	*oz.*	*pwt.*	*gr.*
47	10	13	12
			63

(16.)

bu.	*p.*	*qt.*	*pt.*
13	2	4	1
			64

(17.)

bu.	*p.*	*qt.*	*pt.*
21	2	6	1
			66

(18.)

bu.	*p.*	*qt.*	*pt.*
13	0	4	0
			72

(19.)

bar.	*gal.*	*qt.*	*pt.*	*gi.*
73	14	0	1	2
				77

(20.)

hhd.	*gal.*	*qt.*	*pt.*	*gi.*
25	61	2	1	2
				81

(21.)

de.	*m.*	*fur.*	*rd.*	*yd.*	*ft.*	*in.*	*bc.*
79	57	6	16	4	1	11	2
							84

(22.)

yd.	*ft.*	*in.*	*bc.*
36	1	8	1
			36

(23.)

m.	a.	r.	po.	yd.	ft.	in.
431	136	2	20	4	3	116
						24

(24.)

a.	r.	po.
189	3	28
		21

(25.)

C.	ft.	in.
114	112	860
		48

(26.)

C.	ft.
9	120
	16

(27.)

T.	ft.
41	18
	18

(28.)

yd.	qr.	na.
55	2	3
		28

(29.)

E.E.	qr.	na.
14	1	3
		56

(30.)

E.Fr.	qr.	na.
44	3	3
		36

(31.)

S.	°	′	″
12	10	49	26
			24

(32.)

S.	°	′	″
6	8	48	44
			15

(33.)

°	′	″
9	36	23
		18

(34.)

ba.	bun.	r.	q.	s.
24	2	1	14	16
				22

(35.)

ba.	bun.	r.	q.	s.
36	3	0	16	18
				28

RULE.

Q. When the multiplier exceeds 12, but is not a composite number, or the exact product of any two numbers in the multiplication table, how must you multiply?

A. Multiply by any two numbers whose product comes nearest to the multiplier; then multiply the upper line, or given sum, by the number that remained, and add this product to the last product, or the sum produced by the two factors, and their sum will be the answer.

EXAMPLES

For Theoretical Exercise on a Slate.

1. A drover bought 38 cows of a farmer at $16,37,5 each; how many dollars did he pay for the whole? *Ans.* $622,25,0.

EXPLANATIONS.

In this example, 4 and 9 are the component parts of 36, for 4 times 9 are 36, which is the nearest number to 38 in the multiplication table. You must first multiply by the 4, which will repeat the multiplicand four times;	$ *c. m.*	
	16,37,5	
	4	
	65,50,0	price of 4 cows.
	9	
	589,50,0	price of 36 cows.
	32,75,0	price of 2 cows added.
	622,25,0	price of 38 cows.

that is, it shows the price of four cows; and you must multiply this product, $65,50,0, by 9, which will repeat it nine times; that is, it shows a number nine times as large as the first product of $16,37,5, repeated four times. You must then multiply the first sum, or upper line, by 2, and add this product

to the product $589,50,0, or $16,37,5, thirty-six times repeated, and the two products will show the price of 38 cows at $16,37,5 each, as 36 and 2 are 38.

(2.)
$ *c. m.*
6,33,8
26

(3.)
$ *c. m.*
78,18,6
29

(4.)
$ *c. m.*
108,40,3
34

(5.)
£ *s. d. qr.*
12 5 6 3
34

(6.)
yr. mo. w. d. h. min. sec.
14 11 3 6 17 37 42
47

(7.)
T. cwt. qr. lb. oz. dr.
41 13 2 17 10 12
31

(8.)
℔ ℥ ʒ ℈ *gr.*
41 8 7 1 19
47

(9.)
lb. oz. pwt. gr.
36 3 4 8
57

(10.)
bu. p. qt. pt.
22 2 7 1
46

11.)
bu. p. qt. pt.
11 1 5 1
37

(12.)
T. p. hhd. gal. qt. pt. gi.
12 1 1 57 3 1 3
39

(13.)
yd. ft. in. bc.
27 2 5 2
43

(14.)									(15.)		
dc.	*m.*	*fur.*	*rd.*	*yd.*	*ft.*	*in.*	*bc.*		*yd.*	*qr.*	*na.*
56	54	7	35	3	1	8	1		44	3	3
							67				26

EXAMPLES

For Practical Exercise.

1. A man bought 7 watches for $18,37,5 each; how many dollars did he pay for the whole? *Ans.* $128,62,5.

2. If one pair of boots cost $5,25; what will 24 pairs cost? *Ans.* $126.

3. A merchant bought 36 barrels of flour at $7,37,5 a barrel; how many dollars did he pay for the whole? *Ans.* $265,50.

4. A farmer sold 24 barrels of pork at £6 7*s.* 8*d.* 2*qr.* a barrel; how much did he receive for the whole? *Ans.* £153 5*s.*

5. A merchant bought 84 pairs of shoes at 4*s.* 6*d.* 1*qr.* a pair; how much did he pay for the whole? *Ans.* £18 19*s.* 9*d.*

6. If a horse can travel one mile in 14*min.* 12*sec.*; in what time can he travel 96 miles? *Ans.* 22*h.* 43*min.* 12*sec.*

7. A merchant bought 5 chests of tea, each weighing 3*cwt.* 2*qr.* 9*lb.*; what was the weight of the whole? *Ans.* 17*cwt.* 3*qr.* 17*lb.*

8. A merchant bought 6 hogsheads of sugar, each weighing 8*cwt.* 1*qr.* 18*lb.*; what was the weight of the whole? *Ans.* 50*cwt.* 1*qr.* 24*lb.*

9. A gentleman bought 36 silver spoons, each

weighing $2oz.$ $14pwt.$ $6gr.$; what was the weight of the whole? *Ans.* $8lb.$ $1oz.$ $13pwt.$

10. A farmer has 6 bins of wheat, each containing $53bu.$ $3p.$ $5qt.$ $1pt.$ of wheat; how much is there in all of them? *Ans.* $323bu.$ $2p.$ $1qt.$

11. A gentleman had 36 bottles of wine, each containing $1qt.$ $1pt.$ $3gi.$ of wine; how many gallons had he in all? *Ans.* $16gal.$ $3qt.$ $1pt.$

12. A merchant bought 8 casks of brandy, each containing $41gal.$ $3qt.$ $1pt.$; how many gallons are there in all? *Ans.* $335gal.$

13. If a man travelled 8 days, each day $35m.$ $6fur.$ $30rd.$; how many miles did he travel in all? *Ans.* $286m.$ $6fur.$

14. A gentleman bought 6 farms, each containing $210a.$ $3r.$ $20po.$; how many acres did he buy in all? *Ans.* $1265a.$ $1r.$

15. How many acres are there in 5 lots, each containing $75a.$ $3r.$ $28po.$? *Ans.* $379a.$ $2r.$ $20po.$

16. A man had 7 parcels of wood, each containing $8C.$ $76ft.$; how many cords had he in all? *Ans.* $60C.$ $20ft.$

17. A merchant bought 28 pieces of calico, each containing $35yd.$ $2qr.$ $1na.$; how many yards did he buy in all? *Ans.* $995yd.$ $3qr.$

18. A merchant bought 12 pieces of cloth, each containing $18yd.$ $1qr.$; how many yards did he buy in all? *Ans.* $219yd.$

Note.—To Teachers. The learner should be exercised in a variety of examples, until he has become accustomed to every operation, and is able to multiply any sum without errour.

COMPOUND DIVISION.

Q. What is COMPOUND DIVISION?

A. Compound Division teaches a short way of doing Compound Substraction.

EXAMPLES

For Mental Exercise.

1. James paid twenty-eight cents for eight oranges; how much did he pay for each?

2. William gave thirty-seven cents and five mills for three slates; how much did he pay for each?

3. If you can buy four hats for sixteen dollars and fifty cents; how many dollars is it apiece?

4. John bought four penknives for fifty cents; how many cents did he pay for each?

5. Jane paid three shillings for four yards of riband, how many pence did she pay a yard?

6. Rufus learned six lessons in one hour and thirty minutes; how many minutes was he in learning each lesson?

7. Peter bought one pound and four ounces of raisins, and divided them equally among five boys; how many ounces did each receive?

8. Three boys gathered three pecks and three quarts of chestnuts, and divided them equally; how many quarts did each boy have?

9. A gentleman put one gallon, three quarts, and one pint of wine into five bottles; how many pints were there in each bottle?

10. If you have a stick that is one foot and three inches long, and cut it in three pieces; how many inches long will each piece be?

11. Jane bought two yards and one quarter of riband, and divided it equally between her two sisters and herself; how many quarters did each have?

12. A tailor made six coats of four yards and two quarters of cloth; how many quarters were there in each coat?

Note.—To TEACHERS. The learner should be required to answer, mentally, the preceding questions, and various others of an equally simple nature, before he is required to use a slate.

RULE.

Q. How must the different numbers, or quantities to be divided, be placed in Compound Division?

A. They must be placed with the lowest denomination at the right hand, and the highest at the left, as in Compound Multiplication.

Q. At which hand of the dividend must the divisor be placed?

A. It must be placed at the left of the dividend, as in Simple Division.

Q. Why do you begin at the left hand of the dividend to divide?

A. Because the different denominations decrease in quantity from the left hand to the right, as in Simple Division.

Q. When the divisor does not exceed 12, where must the quotient be placed?

A. It must be placed under the dividend, as in Simple Division.

Q. How must the left hand denomination be divided?

A. The same as in Simple Division.

Q. When there is a remainder, what must be done with it?

A. The remainder, if any, must be brought to, or reduced to the next lower, or inferiour denomination; and, after adding the product to the next lower denomination, the sum must be divided by the divisor, as before: in this manner must each denomination be divided.

Q. If the divisor be not contained in the next lower denomination of the dividend, how must you proceed?

A. A cipher must be written in the quotient, as in Simple Division, and this denomination added to the next lower denomination.

EXPLANATIONS.

As Compound Substraction is the reverse of Compound Addition, so is Compound Division the reverse of Compound Multiplication. You have learned, that Compound Addition is joining, or collecting numbers, or quantities, together of different denominations; and that Compound Substraction is separating numbers of different denominations, or taking them one from the other. You have also learned, that Compound Multiplication is the bringing together of *similar* or *equal* sums. Now you must learn, that Compound Division is separating larger

sums into smaller, the smaller being *equal* one to another; as, one shilling, divided into four equal parts, gives us three pence for each part; and also shows us, that three pence can be substracted from a shilling four times.

EXAMPLES

For Theoretical Exercise on a Slate.

FEDERAL MONEY.

1. A farmer sold 7 bushels of wheat for $7,87,5; how much did he receive for each bushel? *Ans.* $1,12,5.

EXPLANATIONS.

	$ c. *m.*
	7)7,87,5
	1,12,5

Federal money decreases in a tenfold proportion from the left hand to the right, and is, therefore, as you have been told, nearly allied to whole numbers. You must begin with the 7, and say, 7 in 7, 1 time; set down the 1 under the 7, and proceed to the next figure, the cents. Thus, 7 in 8, 1 time and one over; set down the 1 under the 8, the left hand column of cents, and carry the one as ten. Thus, one ten added to the 7 makes 17, and 7 in 17, 2 times and three over; set down the 2 under the 7, the right hand column of cents, and carry the three as so many tens. Thus, three tens added to the 5 make 35, and 7 in 35, 5 times; set down the 5 under the 5, in the place of mills, and then the work is done.

When your sum consists of dollars and cents only, you must call the last remainder, the cents, tens of mills, and divide them as so many tens of mills; and when your sum consists of dollars only, you must

call the last remainder, the dollars, hundreds of cents, and divide them as so many hundreds of cents, and place the quotient at the right hand of the quotient of dollars, and separate the cents and mills from the dollars, in the quotient, by a comma, as before directed.

Thus, you will readily perceive, that there is no difference between the division of federal money and that of whole numbers, except the proper placing of the comma between the dollars, cents, and mills, in the dividend and quotient; that is, you must, if there be mills and cents in the dividend, separate the first figure at the right hand by a comma, and the second and third figures, in like manner, for cents, and the same in the quotient.

PROOF.

The method of proof is the same in Compound Division as in Simple Division.

(2.)	(3.)	(4.)
\$ *c. m.*	\$ *c. m.*	\$ *c. m.*
5) 9,58,4	6) 37,56,9	8) 736,36,1
(5.)	(6.)	(7.)
\$ *c. m.*	\$ *c.*	\$
9) 481,61,4	11) 98,34	12) 81336

STERLING OR ENGLISH MONEY.

1. A gentleman divided £132 9*s.* 9*d.* equally among his 4 sons; how much did each receive? *Ans.* £33 2*s.* 5*d.* 1*qr.*

EXPLANATIONS.

£	s.	d.	qr.
4)132	9	9	0
33	2	5	1

Begin, as directed in the rule, at the left hand, or highest denomination, to divide. Divide the left hand denomination as in Simple Division. Thus, 4 in 13, 3 times and one over; set down the 3, and carry the one to the next right hand figure, the 2, as ten. Thus, ten added to the 2 makes 12, and 4 in 12, 3 times; set down the 3 under the 2, and, as there is no remainder of pounds, you must begin anew with the shillings. Thus, 4 in 9, 2 times and one over; set down the 2 under the 9, in the place of shillings, and carry the one, the remainder of shillings, to the 9, in the column of pence, as twelve, because one shilling is equal to twelve pence, the next lower denomination. Thus, twelve added to the 9, in the column of pence, makes 21, and 4 in 21, 5 times; set down the 5 under the 9, in the place of pence, and carry the one, the remainder of pence, to the cipher, in the column of farthings, as four, because one penny is equal to four farthings, the next lower denomination. Thus, four added to the cipher is 4, and 4 in 4, 1 time; set down the 1 under the cipher, in the place of farthings, and then the work is done.

(2.)	(3.)	(4.)
£ s. d. qr.	£ s. d. qr.	£ s. d. qr.
5)36 10 6 1	6)142 8 7 2	8)189 12 8 0

TIME.

1. Divide 24*yr.* 133*d.* 12*h.* 13*min.* 24*sec.* by 6.
Ans. 4*yr.* 22*d.* 6*h.* 2*min.* 14*sec.*

EXPLANATIONS.

	yr.	*d.*	*h.*	*min.*	*sec.*
6)	24	133	12	13	24
	4	22	6	2	14

Divide, as before directed, the left hand, or highest denomination, as in Simple Division. If there be a remainder of years, it must be carried to the column of days, as so many times three hundred and sixty-five, because one year is equal to three hundred and sixty-five days, the next lower denomination; but in this example there is no remainder of years, and you must, therefore, begin anew to divide the days. Thus, 6 in 13, the first two figures at the left hand of the days, 2 times and one over; set down the 2 under the 3, the second figure, and carry the one to the 3, the third figure of days, and say, 6 in 13, 2 times and one over; set down the 2 under the 3, and carry the one as twenty-four, because one day is equal to twenty-four hours, the next lower denomination. Thus, twenty-four added to 12, in the column of hours, makes 36, and 6 in 36, 6 times; set down 6 under the 12, and as there is no remainder of hours, you must begin anew to divide the minutes. Thus, 6 in 13, 2 times and one over; set down 2 under the 13, and carry the one, the remainder of minutes, to the 24, in the column of seconds, as sixty, because one minute is equal to sixty seconds, the next lower denomination. Thus, sixty added to the 24, in the column of seconds, makes 84, and 6 in 84, 14 times; set down the 14 under the 24, and then the work is done.

When the number of any denomination to be divided is composed of two or more figures, you must divide it the same as in Simple Division, until the last remainder, which must be considered as

so many units as it will take of the next lower denomination to make one of that which you are dividing, as in the present example.

(2.)	(3.)
yr. mo. w. d. h. min. sec.	yr. d. h. min. sec.
7)84 2 1 3 18 53 42	8)32 128 12 19 12

AVOIRDUPOIS WEIGHT.

1. A farmer had 17*cwt.* 16*lb.* 4*oz.* of cheese in 4 boxes; how much was there in each box? *Ans.* 4*cwt.* 1*qr.* 4*lb.* 1*oz.*

EXPLANATIONS.

cwt.	*qr.*	*lb.*	*oz.*	*dr.*
4)17	0	16	4	0
4	1	4	1	0

As before, divide the left hand denomination as in Simple Division. When there is a remainder of hundred-weight, carry it to the quarters as so many times four, because one hundred-weight is equal to four quarters. When there is a remainder of quarters, carry it to the pounds as so many times twenty-eight, because one quarter is equal to twenty-eight pounds. When there is a remainder of pounds, or ounces, carry the remainder to the ounces, or drachms, as the case may be, as so many times sixteen, because one pound is equal to sixteen ounces, and one ounce is equal to sixteen drachms.

(2.)	(3.)
T. cwt. qr. lb. oz. dr.	*cwt. qr. lb. oz. dr.*
6)72 12 1 14 12 6	8)10 2 24 8 8

APOTHECARIES WEIGHT.

1. Divide 24℔ 6℥ 0ʒ 1℈ 16*gr.* by 6. *Ans.* 4℔ 1℥ 0ʒ 0℈ 6*gr.*

EXPLANATIONS.

	℔	℥	ʒ	℈	*gr.*
6)	24	6	0	1	16
	4	1	0	0	6

As before, divide the left hand denomination as in Simple Division. When there is a remainder of pounds, carry it to the ounces as so many times twelve; carry the remainder of ounces to the drachms as so many times eight; carry the remainder of drachms to the scruples as so many times three; carry the remainder of scruples as so many times twenty, to the grains, which, divide, and then the work is done.

In this manner must every sum be worked in Compound Division, when the divisor does not exceed 12. You must, in all cases, carry the remainder to the next lower, or left hand denomination, as so many times the number that it takes of the next lower denomination to make one of the same denomination as the remainder. This is all the difference between Compound and Simple Division, that, in Simple Division, you carry the remainder to the next figure as so many tens; but, in Compound Division, as so many times the number that it takes of the next lower denomination to make one of the remainder.

By paying particular attention to this difference, Compound Division will be rendered very plain to you.

(2.)

	℔	℥	ʒ	℈	*gr.*
8)	14	8	6	2	16

(3.)

	℔	℥	ʒ	℈	*gr.*
12)	49	0	1	0	12

TROY WEIGHT.

(1.)

	lb.	oz.	pwt.	gr.
8)	241	4	17	8

(2.)

	lb.	oz.	pwt.	gr.
4)	133	6	5	4

DRY MEASURE.

(1.)

	bu.	p.	qt.	pt.
4)	17	1	4	0

(2.)

	bu.	p.	qt.	pt.
6)	90	0	6	0

WINE MEASURE.

(1.)

	T.	p.	hhd.	gal.	qt.	pt.	gi.
6)	15	1	1	22	2	1	2

(2.)

	bar.	gal.	qt.	pt.	gi.
4)	84	29	0	1	0

LONG MEASURE.

(1.)

	de.	m.	fur.	rd.	yd.	ft.	in.	bc.
4)	36	44	6	8	4	2	9	1

(2.)

	yd.	ft.	in.	bc.
8)	28	1	6	2

LAND OR SQUARE MEASURE.

(1.)

	m.	a.	r.	po.	yd.	ft.	in.
6)	246	420	3	0	18	6	96

(2.)

	a.	r.	po.
8)	678	2	24

SOLID OR CUBICK MEASURE.

(1.)

	C.	ft.	in.
4)	376	116	1600

(2.)

	T.	ft.
6)	49	26

(3.)

	C.	ft.
8)	169	0

CLOTH MEASURE.

(1.)	(2.)	(3.)
yd. qr. na.	*E.E. qr. na.*	*E.Fr. qr. na.*
4)86 1 0	8)48 4 0	12)50 2 0

CIRCULAR MOTION.

(1.)	(2.)	(3.)
S. ° ′ ″	*S.* ° ′ ″	° ′ ″
6)18 12 49 12	8)24 16 49 4	12)169 12 48

PAPER.

(1.)	(2.)
ba. bun. r. q. s.	*ba. bun. r. q. s.*
4)56 4 1 12 20	6)14 2 1 15 12

RULE.

Q. When the divisor is a composite number, that is, when any two numbers in the division table, being multiplied together, will produce a number exactly equal to the divisor, how must you divide?

A. The dividend must first be divided by one of those numbers, or component parts, then that quotient by the other number, and the last quotient will be the answer, as in Simple Division.

EXAMPLES

For Theoretical Exercise on a Slate.

1. A farmer sold 28 cows for $448,84; how many dollars did he receive for each cow? *Ans.* $16,03.

EXPLANATIONS.

Here, in this example, 4 and 7 are the component parts of 28, for 4 times 7 are 28. You must divide by the 4, and the quotient will be $112,21. You must then divide that quotient by 7, and the next quotient will be $16,03. The principle of this operation is very plain; for in the first division you divide by 4, assuming the position, that 4 cows were sold for $448,84; which sum, when separated into four equal parts, shows the price of each cow to be $112,21; but as there were 28, instead of 4, which is 4 times 7, you then divide the $112,21 into 7 parts, which then shows you the $448,84 divided, or separated, into 28 equal parts, each of which part is $16,03.

```
   $ c.
4)448,84
  ------
7)112,21
  ------
   16,03
```

(2.)

```
     $  c. m.
24)749,44,8
   --------
```

(3.)

```
     $  c. m.
36)535,75,2
   --------
```

(4.)

```
     $  c. m.
48)813,36,0
   --------
```

(5.)

```
     $  c. m.
32)964,54,4
   --------
```

(6.)

```
     £   s. d. qr.
15)732  11  6  3
   -------------
```

(7.)

```
     £   s. d. qr.
18)823   8  6  0
   -------------
```

(8.)

```
   yr. mo. w. d. h. min. sec.
16)84   2   1    3  17  37  36
   ---------------------------
```

(9.)

```
   T. cwt. qr. lb. oz. dr.
24)96  6    3   24  6   0
   ----------------------
```

(10.)

	bu.	*p.*	*qt.*	*pt.*
36)	426	2	6	1

(11.)

	hhd.	*gal*	*qt.*	*pt.*	*gi.*
64)	860	24	2	1	2

(12.)

	m.	*fur.*	*rd.*	*yd.*	*ft.*	*in.*	*bc.*
42)	786	6	30	5	1	10	2

(13.)

	a.	*r.*	*po.*	*yd.*	*ft.*	*in.*
81)	462	2	16	30	3	136

(14.)

	C.	*ft.*	*in.*
36)	489	76	860

(15.)

	yd.	*qr.*	*na.*
48)	252	2	2

RULE.

Q. When the divisor is large, and not a composite number, how must you divide?

A. The dividend must be divided by the whole divisor, as in Simple Division.

EXAMPLES

For Theoretical Exercise on a Slate.

1. A man divided \$107,62,5 equally among 205 men; how much did each man receive? *Ans.* \$0,52,5.

EXPLANATIONS.

```
       $   c. m.
205 ) 107,62,5 ( 0,52,5
      1025
      ----
        512
        410
        ----
        1025
        1025
```

Divide as in Simple Division. Then, if the dividend consists of dollars, cents, and mills, separate the figure at the right hand, in the quotient, by a comma, for the mills, and the next two figures for cents, and then the work is done. Thus, in this ex-

[illegible], you must separate 5, at the right hand, for mills, and the 52 for cents; and as the number of dollars, 107, was less than the number of men, the divisor, 205, you must set a cipher in the place of dollars. You must always remember, that, when the dividend is composed of dollars, cents, and mills, there must be one figure separated at the right hand, in the quotient, for mills, and the next two for cents, and the remainder at the left hand will be dollars. When the dividend consists of dollars and cents only, you must, if you have a remainder of cents, add a cipher at the right of the remainder, and divide the amount, and the quotient will be mills.

By paying attention to these, and the preceding EXPLANATIONS, you will be able to work any sum in federal money.

(2.)	(3.)	(4.)
$ *c. m.*	$ *c. m.*	$ *c.*
95) 735,68,0	35) 58,80,0	46) 93,38
(5.)	(6.)	(7.)
$ *c. m.*	$ *c. m.*	$
37) 416,21,3	49) 516,36,2	78) 8346
(8.)	(9.)	(10.)
c. m.	*c. m.*	*c. m.*
58) 34,8	67) 33,5	83) 91,3

11. A gentleman divided £376 18*s*. 9*d*. 2*qr*. equally among 29 men; how much did each man receive? *Ans.* £12 19*s*. 11*d*. 2*qr*.

EXPLANATIONS.

Divide the pounds, or highest denomination, as in Simple Division. Then multiply the remainder of pounds by 20, and add in the 18 shillings, which are in the dividend, and divide the amount; set down the quotient of shillings, the 19, at the right hand of the pounds. Multiply the remainder of shillings by 12, and add in the 9 pence, which are in the dividend, and divide the amount; set down the quotient of pence, the 11, at the right hand of the shillings. Multiply the remainder of pence by 4, and add in the 2 farthings, which are in the dividend, and divide the amount; set down the quotient of farthings, at the right hand of the pence, and then the work is done.

```
     £   s. d. qr.   £  s. d. qr.
29)376  18  9  2(12 19 11 2
   29
   —
    86
    58
    —
    28 pounds remaining.
     20
    ——
    578( 19
    29
    ——
    288
    261
    ——
     27 shillings remaining
     12
    ——
    333( 11
    29
    ——
     43
     29
    ——
     14 pence remaining.
      4
    ——
     58(2
     58
```

In this manner must every sum be worked in Com-

pound Division when the divisor is a large number, but not a composite number. You must, in every case, divide the highest, or left hand denomination, as in Simple Division. When there is a remainder, as in the present example, multiply it by the number that it takes of the next lower denomination to make one in that of the remainder, and add in all of the next lower denomination which there is in the dividend. Proceed in this manner with every number in each denomination.

(12.)

	£	*s.*	*d.*	*qr.*
23)	973	15	7	0

(13.)

	yr.	*mo.*	*w.*	*d.*	*h.*	*min.*	*sec.*
34)	123	7	2	5	18	56	14

(14.)

	T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
37)	524	17	2	3	14	13

(15.)

	℔	℥	ʒ	℈	*gr.*
41)	167	9	5	1	18

(16.)

	lb.	*oz.*	*pwt.*	*gr.*
51)	84	7	16	19

(17.)

	bu.	*p.*	*qt.*	*pt.*
57)	384	2	6	1

(18.)

	hhd.	*gal.*	*qt.*	*pt.*	*gi.*
62)	648	46	3	1	3

(19.)

	yd.	*ft.*	*in.*	*bc.*
67)	169	2	9	2

(20.)

	a.	*r.*	*po.*	*yd.*	*ft.*	*in.*
73)	9723	3	37	5	5	14

(21.)

	yd.	*qr.*	*na.*
85)	976	3	2

EXAMPLES

For Practical Exercise.

1. If you buy a cheese, weighing 64 pounds, for \$5,12; how many cents do you pay a pound? *Ans.* \$0,08.

2. A sea captain divided \$17500,75 equally among 125 sailors; how many dollars did each sailor receive? *Ans.* \$140,00,6.

3. A farmer sold 125 bushels of wheat for \$190; how much did he receive for each bushel? *Ans.* \$1,52.

4. If you pay £1 2*s.* 8*d.* 2*qr.* for 5 bushels of wheat; how much is that a bushel? *Ans.* 4*s.* 6*d.* 2*qr.*

5. A merchant bought 81 barrels of flour for £147 16*s.* 6*d.*; how much did he pay a barrel? *Ans.* £1 16*s.* 6*d.*

6. A man walked 36 miles in 16*h.* 30*min.* 36*sec.*; how long was he walking each mile? *Ans.* 27*min.* 31*sec.*

7. A merchant bought 37 hogsheads of tobacco, weighing 15*T.* 12*cwt.* 0*qr.* 21*lb.*; what was the weight of each hogshead? *Ans.* 8*cwt.* 1*qr.* 21*lb.*

8. A merchant bought 109 hogsheads of sugar, weighing 50*T.* 19*cwt.* 3*qr.* 20*lb.*; what was the weight of each hogshead? *Ans.* 9*cwt.* 1*qr.* 12*lb.*

9. A gentleman bought 12 silver spoons, weighing 3*lb.* 2*oz.* 13*pwt.* 12*gr.*; what was the weight of each spoon? *Ans.* 3*oz.* 4*pwt.* 11*gr.*

10. A farmer has 4 bins of wheat, containing 486*bu.* 2*p.* 4*qt.* of wheat; how much was there in each bin? *Ans.* 121*bu.* 2*p.* 5*qt.*

11. A man travelled 52*m.* 4*fur.* 21*rd.* in 21 hours; how many miles did he travel in each hour? *Ans.* 2*m.* 4*fur.* 1*rd.*

12. A company of 15 men bought 4509*a.* 1*r.* 20*po.* of land, and paid an equal proportion; how many acres was each man's share? *Ans.* 300*a.* 2*r.* 20*po.*

13. A merchant bought 8 pieces of cloth, containing 498*yd.* 2*qr.*; how many yards were there in each piece? *Ans.* 62*yd.* 1*qr.* 1*na.*

14. A paper maker had 969*ba.* 4*bun.* 1*r.* of paper in 4 boxes; how many bales were there in each box? *Ans.* 242*ba.* 2*bun.* 0*r.* 15*q.*

EXPLANATIONS.

You have now, I trust, become fully acquainted with the working of figures in the fundamental rules of Arithmetick, both Simple and Compound, that is, of WHOLE NUMBERS, or INTEGERS. I shall now treat of PARTS of these *whole numbers*, or *integers*, or, as they are generally called, FRACTIONS.

FRACTION is, therefore, used to describe a part, merely, of any thing that may be the subject of consideration. Thus, if we speak of certain weights, as, three ounces and a quarter, (that is, a quarter of an ounce,) this quarter being but a *part*, is called a *fraction;* and if we speak of seven pounds and a quarter, then this "quarter" is also a *fraction:* only observe, that, meaning, as it would, a quarter of a pound, so it would be called a fraction of a pound, while the other means a fraction of an ounce.

Thus, PARTS of any thing, whether of weights, of measures, of money, or of periods of time; parts of every size, as, halves, quarters, thirds, one fourth, three fourths, four fifths, seven eighths; or, in short, any other conceivable quantity, either small or large, as, a thousandth part, or, as nine hundred and ninety-nine such parts, or any portion short of whole, is a fraction; and the treatment, or the working of these parts of numbers, is called the working of fractions; while, in order to distinguish them from these FRACTIONS, the numbers of which we have heretofore treated, are called INTEGERS, or WHOLE NUMBERS.

FRACTIONS.

Q. What are FRACTIONS?

A. Fractions, or broken numbers, are the *parts* of a whole number, or integer, as, *parts* of a pound, yard, mile, &c.

Q. How many kinds of fractions are there?

A. Two; Vulgar and Decimal Fractions.

VULGAR FRACTIONS.

Q. What is a Vulgar Fraction?

A. A Vulgar Fraction is a part of a unit, or integer, expressed, or represented, by two numbers, one placed directly above the other, with a line between them; thus, $\frac{3}{4}$ signifies three fourths of one, and $\frac{2}{3}$ signifies two thirds of one, &c.

Q. What is the number above the line called?

A. It is called the numerator.

Q. Why is it called the numerator?

A. Because it shows the number of parts the fraction contains.

Q. What is the number below the line called?

A. It is called the denominator.

Q. Why is it called the denominator?

A. Because it shows the *quantity* of these parts, or it shows into how many parts a unit, or whole number, is divided

Vulgar Fractions are either proper, improper, compound, or mixed.

Q. What is a *proper* or *simple* Vulgar Fraction?

A. A proper, or simple fraction, is when the numerator is less than the denominator, as, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{5}$, &c.

Q. What is an *improper* fraction?

A. An improper fraction is when the numerator exceeds the denominator, as, $\frac{8}{3}$, $\frac{5}{4}$, $\frac{12}{4}$, &c.

Q. What is a *compound* fraction?

A. A compound fraction is a fraction of a fraction, connected or coupled by the word *of*, as, $\frac{2}{3}$ of $\frac{1}{12}$, $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$, &c.

Q. What is a *mixed* number?

A. A mixed number is composed of a whole number and a fraction, as, $4\frac{1}{3}$, $16\frac{3}{12}$, &c.

A whole number may be expressed like a fraction, by drawing a line under it, and placing 1 below for a denominator, as, $7=\frac{7}{1}$, and $14=\frac{14}{1}$, &c.

RULE.

Q. How must you reduce, or abbreviate fractions to their lowest terms?

A. The numerator and denominator of the given fraction must be divided by any number that will divide them without a remainder, and the quotient again in the same manner, and so on, till it appears that there is no number greater than 1 that will divide them, and the last quotient will express the given fraction in its lowest or least term.

EXAMPLES

For Theoretical Exercise on a Slate.

1. Reduce $\frac{24}{48}$ to its lowest terms. *Ans.* $\frac{1}{2}$.

EXPLANATIONS

You must first set down the fraction, and divide the numerator by 6, and say, 6 in 24 four times; set down the 4 for the

$$6)\frac{24}{48}=\frac{4}{8}\;4)=\frac{1}{2}\ \textit{Ans.}$$

or thus,

$$12)\frac{24}{48}=\frac{2}{4}=\frac{1}{2}\ \textit{Ans.}$$

numerator of a new fraction: you must then divide the denominator by 6, and say, 6 in 48 eight times; set down the 8, for the denominator, under the 4, the new numerator. You must then divide the new fraction by 4, which will give $\frac{1}{2}$, the lowest fraction, which is the same in value of $\frac{24}{48}$. You will readily perceive, that you do not alter the value of the fraction by this operation, for the numerator of the quotient bears the same proportion to the denominator of the quotient in each place, that the numerator of the dividend bears to the denominator of the dividend, as 24 is the half of 48, 4 is the half of 8, and 1 is the half of 2. You must also bear in mind, that it does not make any difference what number you take for a divisor, if it will divide the terms of the fraction without a remainder.

You will readily perceive, that dividing the numerator and denominator both by the same number, does not, in any case, alter the value of a fraction. Thus, $\frac{24}{48}$ is equal to $\frac{4}{8}$, and $\frac{4}{8}$ is equal to $\frac{1}{2}$, and, therefore, $\frac{1}{2}$ is equal to $\frac{24}{48}$. Thus you see, that the operation only alters the terms of the fraction, and not its value.

2. Reduce $\frac{216}{288}$ to its lowest terms. *Ans.* $\frac{3}{4}$.
3. Reduce $\frac{192}{576}$ to its lowest terms. *Ans.* $\frac{1}{3}$.
4. Reduce $\frac{144}{216}$ to its lowest terms. *Ans.* $\frac{2}{3}$.
5. Reduce $\frac{45}{315}$ to its lowest terms. *Ans.* $\frac{1}{7}$.
6. Reduce $\frac{32}{256}$ to its lowest terms. *Ans.* $\frac{1}{8}$.
7. Reduce $\frac{525}{630}$ to its lowest terms. *Ans.* $\frac{5}{6}$.
8. Reduce $\frac{66}{72}$ to its lowest terms. *Ans.* $\frac{11}{12}$.
9. Reduce $\frac{171}{180}$ to its lowest terms. *Ans.* $\frac{19}{20}$.
10. Abbreviate $\frac{784}{952}$ as much as possible. *Ans.* $\frac{14}{17}$.

RULE.

Q. How do you find the value or quantity of a fraction in the known parts of the integer, that is, in the inferiour denomination of the integer as to coin, weight, measure, &c.?

A. The numerator must be multiplied by the common parts of the integer, and the product must be

divided by the denominator; and if there be a remainder, it must be reduced to the next inferiour denomination, and the product divided again, as before, till it is reduced to the lowest denomination, or till there is no more remainder.

EXAMPLES

For Theoretical Exercise on a Slate.

1. What is the value of $\frac{2}{5}$ of a pound sterling? *Ans.* 8s.

EXPLANATIONS.

$$\begin{array}{r} £ \\ 2 \\ 20 \\ \hline 5)40 \\ \hline 8\ s. \end{array}$$

In this example, you must multiply the 2, the numerator, by 20, because twenty shillings make a pound, and, also, because shilling is the next inferiour denomination below pound, in sterling money. When there is a remainder of shillings, you must multiply it by 12, because twelve pence make one shilling, and pence are the next inferiour denomination; and if there be a remainder of pence, multiply it by 4, because four farthings make one penny. The denominator, 5, shows that a pound is divided into five parts; and the numerator, 2, shows how many of those parts the fraction contains. You multiply the 2 by 20, because it is two parts of twenty, of which you wish to find the amount, and 5 is the amount of each of the 2 parts. If you wish to obtain the amount of only one part, you would divide by 5, without multiplying the 2 by 20, so you must increase the 20 as many times as the numerator expresses. Again, it is very obvious, that if £1, or 20s., is divided into 5 parts, that one part must be 4 shillings, because four shillings is the fifth part of £1, or 20; and the numerator, in this example, expresses two parts, consequently, the value of the fraction is 8 shillings, as two times 4 shillings make 8 shillings.

2. What is the value of $\frac{2}{3}$ of a day? *Ans.* 16h.

EXPLANATIONS.

In this example, you must multiply the 2, the numerator, by 24, because twenty-four hours make a day, and also because hour is the next inferiour denomination in time. When there is a remainder of hours or minutes, you must multiply it by 60, because sixty minutes make one hour, and sixty seconds one minute. Thus, then, it is very plain, that when you wish to continue the division, and express the fraction in the inferiour denomination, you must reduce the remainder to the next inferiour denomination by multiplying the remainder, as before directed, and divide by the denominator of the fraction.

$$\begin{array}{r} 2 \\ 24 \\ \hline 3)48 \\ \hline 16h. \end{array}$$

3. What is the weight of $\frac{3}{4}$ of a pound avoirdupois? *Ans.* 12*oz.*

EXPLANATIONS.

The value of any fraction may also be found by dividing the number that it takes of the next inferiour denomination to make a whole number of the denomination of the given fraction, by the denominator of the fraction, and multiply the product by the numerator of the fraction. Thus, in this example, sixteen ounces make one pound, and you must divide the 16 by 4, and then multiply the product, which is 4 also, by 3, the numerator, which gives the answer, 12*oz.*, the same as by the other operation. If there be a remainder, proceed as with the first division, namely, divide the number that it takes of the next lower denomination to make one of the remainder.

$$\begin{array}{l} 4)16oz. = \text{to } 1lb. \\ \hline \quad 4 \\ \quad 3 \\ \hline \quad 12oz. \end{array}$$

4. What is the value of $\frac{9}{13}$ of a day? *Ans.* 16*h.* 36*min.* 55$\frac{5}{13}$ *sec.*

5. What is the value of $\frac{7}{9}$ of a hundred-weight? *Ans.* 3*qr.* 3*lb.* 1*oz.* 12$\frac{4}{9}$*dr.*

6. What is the value of $\frac{3}{5}$ of a pound troy? *Ans.* 7*oz.* 4*pwt.*

7. What is the value of $\frac{3}{8}$ of a bushel? *Ans.* 12*qt.*

8. What is the value of $\frac{6}{7}$ of a hogshead of wine? *Ans.* 54*gal.*

9. What is the value of $\frac{5}{6}$ of a mile? *Ans.* 6*fur.* 26*po.* 11*ft.*

10. What is the value of $\frac{5}{8}$ of an acre? *Ans.* 2*r.* 20*po.*

11. What is the value of $\frac{2}{3}$ of a yard? *Ans.* 2*qr.* $2\frac{2}{3}$*na.*

12. What is the value of $\frac{7}{8}$ of an ell English? *Ans.* 4*qr.* $1\frac{1}{2}$*na.*

RULE.

Q. How do you reduce any given quantity or inferiour denomination to the fraction of some superiour denomination, which shall retain the same value?

A. The given sum must be reduced to the lowest denomination mentioned for a numerator; then the unit must be reduced to the same denomination for a denominator, which will be the fraction required.

EXAMPLES

For Theoretical Exercise on a Slate.

1. Reduce 13*s.* 4*d.* to the fraction of a pound. *Ans.* £$\frac{2}{3}$.

EXPLANATIONS.

You must first reduce 13*s.* 4*d.* to pence, for a numerator, by multiplying the 13 by 12, and adding in the 4 pence, because pence is the lowest denomination mentioned in the given sum, 13*s.* 4*d.* You must then reduce £1 or 20*s.* to pence, for a denominator, and you will find that a pound, reduced to pence, is divided into 240 parts, and the numerator contains 160 similar parts, because both numerator and denominator express pence; hence, the denominator,

s.	*d.*	
13	4	*given sum.*
12		
160		$8)\frac{160}{240} = 5)\frac{20}{30} = 2)\frac{4}{6} = \frac{2}{3}$ *Ans.*

s.	
20	*integral part.*
12	
240	

240, shows that a unit, or £1, is divided into 240 parts, and the numerator shows that the fraction contains 160 of those parts, which, reduced to its lowest given term, gives $\frac{2}{3}$ of a pound.

2. Reduce 4*d.* 2*qr.* to the fraction of a shilling. *Ans.* $\frac{3}{8}$.

3. Reduce 6 hours to the fraction of a day. *Ans.* $\frac{1}{4}$.

4. Reduce 13*cwt.* 3*qr.* 20*lb.* to the fraction of a tun. *Ans.* $\frac{39}{56}$.

5. Reduce 7*oz.* 4*pwt.* to the fraction of a pound troy. *Ans.* $\frac{3}{5}$.

6. Reduce 12*qt.* to the fraction of a bushel. *Ans.* $\frac{3}{8}$.

7. Reduce 1*hhd.* 49*gal.* of wine to the fraction of a tun. *Ans.* $\frac{4}{9}$.

8. Reduce 9*gal.* to the fraction of a hogshead. *Ans.* $\frac{1}{7}$.

9. Reduce 6*fur.* 16*po.* to the fraction of a mile. *Ans.* $\frac{4}{5}$.

10. Reduce 2*r.* 20*po.* to the fraction of an acre. *Ans.* $\frac{5}{8}$.

11. Reduce 3*qr.* 3*na.* to the fraction of a yard. *Ans.* $\frac{15}{16}$.

12. Reduce 4*qr.* 1½*na.* to the fraction of an ell English. *Ans.* $\frac{7}{8}$.

RULE.

Q. How do you reduce an improper fraction to a whole or mixed number?

A. The numerator must be divided by the denominator, and the quotient will be the answer sought in a whole or mixed number.

EXAMPLES

For Theoretical Exercise on a Slate.

1. Reduce $\frac{72}{6}$ to its equivalent whole, or mixed number. *Ans.* 12.

EXPLANATIONS.

Here, in this example, you divide 72, the numerator, by the 6, the denominator. The numerator shows how many parts the fraction contains, and the denominator shows how many of those parts it requires to make a unit.

$$\begin{array}{r} 6)72 \\ \hline 12 \end{array}$$

2. Reduce $\frac{933}{11}$ to its equivalent whole, or mixed number. *Ans.* $84\frac{9}{11}$.

3. Reduce $\frac{81}{9}$ to its equivalent whole, or mixed number. *Ans.* 9.

4. Reduce $\frac{749}{17}$ to its equivalent whole, or mixed number. *Ans.* $44\frac{1}{17}$.

RULE.

Q. How do you reduce a mixed number to its equivalent improper fraction?

A. Multiply the integer, or whole number, by the denominator of the fraction, and add the numerator to the product; then that sum must be placed above the denominator for the fraction required.

EXAMPLES

For Theoretical Exercise on a Slate.

1. Reduce $12\frac{7}{9}$ to its equivalent improper fraction. *Ans.* $\frac{115}{9}$.

EXPLANATIONS.

In this example, you must multiply 12 by 9, and add the numerator, the 7, to the product, 108; the sum, 115, is the new numerator of the fraction sought, and 9 the denominator; thus, you will have $\frac{115}{9}$ the improper fraction, equal to $12\frac{7}{9}$. This operation is very plain; for by multiplying the 12 by 9, the denominator, you reduce the 12 to ninths, that is, 108 ninths, and the 7 ninths, added to these, make 115 ninths, the answer.

$12\frac{7}{9}$	
9	denominator.
108	
7	numerator added.
115	new numerator.
9	denominator.

2. Reduce $36\frac{5}{8}$ to its equivalent improper fraction. *Ans.* $\frac{293}{8}$.

3. Reduce $19\frac{12}{18}$ to its equivalent improper fraction. *Ans.* $\frac{354}{18}$.

4. Reduce $54\frac{11}{12}$ to its equivalent improper fraction. *Ans.* $\frac{659}{12}$.

RULE.

Q. How do you reduce a whole number to an equivalent fraction, having a given denominator?

A. The whole number must be multiplied by the given denominator, and the product must then be placed over the said denominator, and it will form the fraction required.

EXAMPLES

For Theoretical Exercise on a Slate.

1. Reduce 6 to a fraction, whose denominator shall be 8. *Ans.* $\frac{48}{8}$.

EXPLANATIONS.

Here, in this example, you must multiply the 6 by the 8, and take the product, 48, for the numerator, and the 8 for the denominator.

$$\begin{array}{r} 6 \\ 8 \\ \hline 48 \\ \hline 8 \end{array}$$

2. Reduce 18 to a fraction, whose denominator shall be 12. *Ans.* $\frac{216}{12}$.

3. Reduce 29 to a fraction, whose denominator shall be 15. *Ans.* $\frac{435}{15}$.

4. Reduce 9 to a fraction, whose denominator shall be 7. *Ans.* $\frac{63}{7}$.

RULE.

Q. How do you reduce a compound fraction to a simple or improper fraction?

A. All the numerators must be multiplied together for a new numerator, and all the denominators must be multiplied together for a new denominator, and they will form the fraction required.

EXAMPLES

For Theoretical Exercise on a Slate.

1. Reduce $\frac{5}{9}$ of $\frac{4}{8}$ of $\frac{3}{4}$ to a simple fraction. *Ans.* $\frac{60}{288}=\frac{5}{24}$.

EXPLANATIONS.

In this example, you must multiply the 5, 4, and 3, the numerators, together, for a new numerator, and the 9, 8, and 4, the denominators, for a new denominator.

5	9
4	8
—	—
20	72
3	4
—	—
60 numerator.	288 denominator.

2. Reduce $\frac{5}{8}$ of $\frac{6}{7}$ of $\frac{15}{19}$ to a simple fraction. *Ans.* $\frac{450}{1064}$.
3. Reduce $\frac{4}{5}$ of $\frac{5}{8}$ of $\frac{7}{9}$ to a simple fraction. *Ans.* $\frac{140}{360}=\frac{7}{18}$.
4. Reduce $\frac{4}{5}$ of $\frac{11}{12}$ of $\frac{19}{25}$ to a simple fraction. *Ans.* $\frac{836}{1500}$.

RULE.

Q. How do you reduce fractions of different denominators to equivalent fractions having a common denominator?

A. Each numerator must be multiplied by all the denominators, except its own, for the new numerator; and all the denominators must be multiplied together for a common denominator.

EXAMPLES

For Theoretical Exercise on a Slate.

1. Reduce $\frac{4}{5}$ and $\frac{5}{7}$ to equivalent fractions, having a common denominator. *Ans.* $\frac{28}{35}$, $\frac{25}{35}$.

EXPLANATIONS.

9	9	7
7	2	5
63	18	35

In this example, you must multiply the 9 and the 7, the denominators, together for a common denominator; and then multiply the denominator, 9, and the numerator, 2, and the 7 by the 5, for new numerators. You will readily perceive, that the value of the fraction is not altered, for each numerator and its denominator is multiplied by the same numbers, and by reducing the new fractions to their lowest terms, you would again have the given fractions.

$\frac{18}{63}$, $\frac{35}{63}$ *Ans.*

2. Reduce $\frac{2}{9}$, $\frac{3}{4}$, and $\frac{5}{8}$ of $\frac{11}{12}$ to equivalent fractions, having a common denominator. *Ans.* $\frac{768}{3456}$, $\frac{2592}{3456}$, $\frac{1980}{3456}$.

3. Reduce $\frac{11}{16}$, $\frac{3}{4}$ of $1\frac{1}{2}$, $\frac{7}{12}$, and $\frac{5}{8}$ to equivalent fractions, having a common denominator. *Ans.* $\frac{8448}{11520}$, $\frac{12960}{11520}$, $\frac{6720}{11520}$, $\frac{7200}{11520}$.

4. Reduce $\frac{4}{5}$, $\frac{6}{20}$, and $\frac{4}{10}$ to equivalent fractions, having a common denominator. *Ans.* $\frac{800}{1000}$, $\frac{300}{1000}$, $\frac{400}{1000}$.

ADDITION OF VULGAR FRACTIONS.

Q. What is Addition of Vulgar Fractions?

A. Addition of Vulgar Fractions teaches to join, or add, several broken numbers, or integers, into one sum.

RULE.

Q. How do you add Vulgar Fractions?

A. Compound fractions must first be reduced to single ones, mixed numbers to improper fractions, and fractions of different integers to those of the same. and all of them to a common denominator;

then the sum of the numerators written over the common denominator, will be the sum of the fractions required.

EXAMPLES

For Theoretical Exercise on a Slate.

1. What is the sum of $\frac{2}{5}$ and $\frac{3}{8}$? *Ans.* $\frac{31}{40}$.

EXPLANATIONS.

8	5
2	3
—	—
16 numerator of $\frac{2}{5}$.	15 numerator of $\frac{3}{8}$.

16
15
—
31 sum of the numerators.

8
5
—
40 common denominator.

Here, in this example, you first multiply the 2 and 8 together, making 16 for the numerator of $\frac{2}{5}$; and then 5 and 3 together, making 15 for the numerator of $\frac{3}{8}$; and then add the 16 and 15, the numerators, for a numerator, making 31; and then you multiply the 5 and 8 together for a common denominator, and you then have the fraction $\frac{31}{40}$. Fractions are quite dissimilar before they are reduced to a common denominator: thus, in the first fraction, $\frac{2}{5}$, a unit is divided into 5 parts; and in the second fraction, $\frac{3}{8}$, a unit is divided into 8 parts; and, therefore, the parts are unequal till reduced to a common denominator, and we then have $\frac{16}{40}$ and $\frac{15}{40}$, which make $\frac{31}{40}$. That the value of the fraction is not altered by the operation is very plain, as the numerator and denominator are multiplied by the same number: hence, it is evident, that a unit in each fraction is divided into 40 parts, and, therefore, the numerators may be added, as they are parts of one common integer.

2. Add $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$ together. *Ans.* $1\frac{7}{8}$

3. Add $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ together. *Ans.* $1\frac{11}{12}$.
4. Add $\frac{6}{9}$, $\frac{5}{8}$, and $\frac{3}{4}$ together. *Ans.* $2\frac{2}{48}$.
5. Add $7\frac{3}{4}$, and $5\frac{2}{3}$ together. *Ans.* $13\frac{5}{12}$.
6. Add $8\frac{5}{8}$ and $11\frac{3}{8}$ together. *Ans.* 20.
7. Add $\frac{3}{4}$*s.* and £$\frac{7}{8}$ together. *Ans.* £$\frac{584}{640}$ = 18*s*, 3*d.*
8. Add $\frac{1}{4}$ of a week, $\frac{1}{3}$ of a day, $\frac{1}{2}$ of an hour, and $\frac{3}{4}$ of a minute together. *Ans.* 2*d.* 2*h.* 30*min.* 45*sec.*
9. Add $\frac{4}{9}$ of a tun to $\frac{5}{12}$ of a *cwt.* *Ans.* 9*cwt.* 1*qr.* $6\frac{2}{9}$*lb.*
10. Add $\frac{1}{5}$ of a pound troy to $\frac{1}{8}$ of an ounce. *Ans.* 2*oz.* $10\frac{1}{2}$*pwt.*
11. Add $\frac{3}{4}$ of a mile to $\frac{7}{10}$ of a furlong. *Ans.* 6*fur.* 28*po.*
12. Add $\frac{7}{8}$ of a mile, $\frac{2}{3}$ of a yard, and $\frac{3}{4}$ of a foot together. *Ans.* 1540*yd.* 2*ft.* 9*in.*

SUBSTRACTION OF VULGAR FRACTIONS.

Q. What is Substraction of Vulgar Fractions?

A. Substraction of Vulgar Fractions teaches to take one broken number, or integer, from another.

RULE.

Q. How do you substract Vulgar Fractions?

A. The fractions must first be prepared as in Addition; one numerator must then be substracted from the other, and the difference placed over the common denominator, will give the difference of the fraction required.

EXAMPLES

For Theoretical Exercise on a Slate.

1. What is the difference between $\frac{6}{8}$ and $\frac{2}{3}$? *Ans.* [illegible]

EXPLANATIONS.

In this example, you substract 2 from 6, which leaves 4, the difference between the two numerators. Here it is plain, that the difference between $\frac{6}{8}$ and $\frac{2}{8}$ is $\frac{4}{8}$ or $\frac{1}{2}$, for the fractions have a common denominator; that is, a unit in each fraction is divided into eight equal parts, and, therefore, the difference of the numerators placed over the common denominator, must express the difference of the fractions.

6
2
—
4 difference between numerators.
$\frac{4}{8}$ = to $\frac{1}{2}$ *Ans.*

2. From $\frac{7}{8}$ take $\frac{2}{3}$. *Ans.* $\frac{5}{24}$.
3. From 14 take $\frac{13}{15}$. *Ans.* $13\frac{2}{15}$.
4. From 365 take $\frac{11}{15}$. *Ans.* $364\frac{4}{15}$.
5. From $13\frac{1}{3}$ take $\frac{3}{4}$ of 15. *Ans.* $2\frac{1}{12}$.
6. Substract $\frac{54}{713}$ from a unit. *Ans.* $\frac{659}{713}$.
7. From £$\frac{1}{3}$ take $\frac{7}{8}$ of a penny. *Ans.* 6*s.* $7\frac{1}{8}$*d.*
8. From 15 days take $9\frac{4}{5}$ days. *Ans.* 5*d.* 4*h.* 48*min.*
9. From $\frac{3}{8}$ of a tun take $\frac{2}{3}$ of $\frac{3}{4}$ of a pound. *Ans.* 7*cwt.* 1*qr.* 27*lb.* 8*oz.*
10. From $\frac{5}{6}$ of a pound troy take $\frac{5}{8}$ of an ounce. *Ans.* 9*oz.* $7\frac{1}{2}$*pwt.*
11. From $\frac{2}{3}$ of a league take $\frac{7}{10}$ of a mile. *Ans.* 1*m.* 2*fur.* 16*po.*
12. From $\frac{3}{5}$ of a league take $\frac{5}{8}$ of a mile. *Ans.* $1\frac{7}{10}$*m.*

MULTIPLICATION OF VULGAR FRACTIONS.

Q. What is Multiplication of Vulgar Fractions?

A. Multiplication of Vulgar Fractions teaches to repeat a whole, or broken number, by a part or the parts of an integer.

RULE.

Q. How do you multiply Vulgar Fractions?

A. All the numerators must be multiplied together for a new numerator, and all the denominators must be multiplied together for a new denominator, which will give the product required. If there be mixed numbers, they must first be reduced to equivalent fractions.

EXAMPLES

For Theoretical Exercise on a Slate.

1. Multiply $\frac{2}{3}$ by $\frac{4}{5}$. *Ans.* $\frac{8}{15}$.

EXPLANATIONS.

In this example, you first multiply the 4 and 2, the numerators, for a new numerator, making 8; and then multiply the 5 and 3, the denominators, for a new denominator, making 15, which make $\frac{8}{15}$, the required

$$\begin{array}{cc} 4 & 5 \\ 2 & 3 \\ \hline 8 & 15 \end{array} \quad \frac{8}{15} \textit{ Ans.}$$

product. Multiplying the denominator of a fraction by any number, is the same as dividing the numerator by the same number. You will readily perceive, that the value of the fraction is increased as many times as the numerator of a fraction is increased; thus, when you multiply the numerator of the fraction, $\frac{2}{3}$ by $\frac{4}{5}$, the fraction is increased four times; but you do not want to increase the value of the fraction four times, but as much less than four as the denominator, 5, indicates; and when you multiply the denominator of the fraction by 5, it makes the value of the fraction five times less; for it takes five times the number of parts to make a unit.

2. Multiply $\frac{3}{8}$ by $\frac{4}{9}$. *Ans.* $\frac{12}{72}=\frac{1}{6}$.
3. Multiply $\frac{2}{7}$ by $\frac{5}{8}$. *Ans.* $\frac{10}{56}=\frac{5}{28}$.
4. Multiply $\frac{5}{8}$ by $\frac{3}{7}$. *Ans.* $\frac{15}{56}$.
5. Multiply $\frac{3}{8}$ by $\frac{4}{5}$. *Ans.* $\frac{12}{40}=\frac{3}{10}$.
6. Multiply $\frac{5}{7}$ by $\frac{9}{11}$. *Ans.* $\frac{45}{77}$.

7. Multiply $5\frac{1}{4}$ by $\frac{1}{6}$. *Ans.* $\frac{7}{8}$.
8. Multiply 20 by $\frac{1}{4}$. *Ans.* 5.
9. Multiply $7\frac{1}{2}$ by $9\frac{1}{4}$. *Ans.* $69\frac{3}{8}$.
10. Multiply 12 by $\frac{3}{4}$ of $\frac{4}{5}$. *Ans.* $7\frac{1}{5}$.
11. Multiply $\frac{13}{18}$ by $\frac{1}{8}$. *Ans.* $\frac{13}{144}$.
12. Multiply $\frac{3}{5}$ of $\frac{1}{2}$ by $\frac{2}{3}$ of $\frac{1}{4}$. *Ans.* $\frac{1}{20}$.

DIVISION OF VULGAR FRACTIONS.

Q. What is Division of Vulgar Fractions?

A. Division of Vulgar Fractions teaches to find how often a part, or the parts, of an integer is contained in a given sum.

RULE.

Q. How do you divide Vulgar Fractions?

A. The fractions must be prepared as in Multiplication; then the divisor must be inverted, and you must proceed as in Multiplication, and the products will be the quotient required.

EXAMPLES

For Theoretical Exercise on a Slate.

1. Divide $\frac{4}{7}$ by $\frac{2}{3}$. *Ans.* $\frac{12}{14}=\frac{6}{7}$.

EXPLANATIONS

In this example, you must first invert the divisor, which will then be $\frac{3}{2}$, and you must then multiply the 4 and the 3 together, and the 2 and the 7 together, which will give the quotient $\frac{12}{14}=\frac{6}{7}$.

$$\begin{array}{cc} 4 & 7 \\ 3 & 2 \\ \hline 12 & 14 \end{array}$$

2. Divide $\frac{5}{7}$ by $\frac{3}{4}$. *Ans.* $\frac{20}{21}$.

3. Divide 4 by $\frac{7}{8}$. *Ans.* $4\frac{4}{7}$.
4. Divide $\frac{17}{21}$ by $\frac{3}{5}$. *Ans.* $1\frac{22}{63}$.
5. Divide 5 by $\frac{7}{10}$. *Ans.* $7\frac{1}{7}$.
6. Divide $\frac{11}{17}$ by $\frac{2}{3}$. *Ans* $\frac{33}{34}$.
7. Divide $\frac{1}{6}$ by $\frac{3}{4}$. *Ans.* $\frac{2}{9}$.
8. Divide $\frac{1}{2}$ of $17\frac{1}{2}$ by $\frac{3}{4}$. *Ans.* $11\frac{2}{3}$.
9. Divide $9\frac{1}{6}$ by $\frac{1}{2}$ of 7. *Ans.* $2\frac{13}{21}$.
10. Divide $7\frac{1}{3}$ by $9\frac{5}{9}$. *Ans.* $\frac{33}{43}$.
11. Divide 9 by $\frac{6}{7}$. *Ans.* $10\frac{1}{2}$.
12. Divide $\frac{2}{3}$ of $\frac{3}{4}$ by $\frac{1}{2}$ of $\frac{2}{3}$. *Ans.* $1\frac{1}{2}$.

DECIMAL FRACTICNS.

Q. What is a DECIMAL FRACTION?

A. A Decimal Fraction is a part of a whole number, or integer, whose denominator is a unit, with a cipher, or ciphers, annexed to it. A Decimal Fraction, however, is usually expressed by writing the numerator only, with a comma, or point, prefixed at the left hand of the fraction; thus, ,5 tenths is the same as $\frac{5}{10}$; and ,25 hundredths is the same as $\frac{25}{100}$, &c.

EXPLANATIONS.

The integer, or whole number, is always divided either into 10, 100, or 1000, &c., equal parts; and, consequently, the denominator of the fraction will always be either 10, 100, 1000, &c., which, being understood, need not be expressed; for the true value of the fraction may be expressed by writing the numerator only with a point before it on the left hand; thus, $\frac{5}{10}$ is written ,5; $\frac{65}{100}$,65, &c. Whole numbers and decimals may be written in the same line, with a point between

them, called the separatrix; thus, $86\frac{4}{10}$ is written 86 ,4; and $9\frac{27}{100}$ is written 9 ,27. You must always remember, that the denominator is repeated in the expression when it is not written; thus, you say, 4 tenths, and 27 hundredths, &c., although you have no denominator expressed in the fraction. Decimals decrease in a tenfold proportion from the left hand to the right, or as they are removed, or recede from the place of units; thus, ,5 is only one tenth of the value which it would express in the place of units, if you should take away the decimal point; and ,05 is only one tenth as much as ,5, and so on.

When ciphers are placed at the right hand of Decimal Fractions, they do not increase, or diminish their value, as every significant figure continues to possess the same value; thus, ,5, ,50, ,500, being $\frac{5}{10}$ five tenth parts, $\frac{50}{100}$ fifty hundredth parts, $\frac{500}{1000}$ five hundred thousandth are all equal in value; for when you annex a cipher to the decimal, the denominator assumes one, consequently, it is multiplying the numerator and denominator by the same number; and, therefore, the proportion between them must ever remain the same. But when ciphers are placed at the left hand of a Decimal Fraction, they diminish the value of the decimal in a tenfold proportion; thus, ,5 ,05 ,005, are the same as $\frac{5}{10}$, $\frac{5}{100}$, $\frac{5}{1000}$, in value, because, in the first example, ,5 shows that a unit is divided into ten parts, and that the fraction contains five of those parts, that is, five tenths; and the second example, ,05, shows that a unit is divided into one hundred parts, and the fraction contains only five of those parts, that is, five hundredths, &c. Hence, it is very evident, that the magnitude of a Decimal Fraction, compared with another, does not depend upon the number of its figures, but upon the value of its first left hand figure.

I presume, from your knowledge of federal money, you will be able to understand this perfectly, for federal money is purely decimal money, of which the dollar is the unit; and the inferiour, or lower denominations, the decimal parts. Thus, 5 dollars and 36 cents are expressed, \$5,36, or \$5, $\frac{36}{100}$. You must remember, that it takes ten tenths, or one hundred cents to make a dollar; and, therefore, when a dollar is divided into one hundred parts, the parts are cents consequently, the ,36 hundredths are cents, of which it takes [illegible]

a unit, or dollar. In federal money, therefore, tenths represent dimes; hundredths represent cents, and thousandths represent mills; but the decimals are commonly expressed, where the unit is a dollar, in cents and mills; or taken together they represent thousandths of a dollar.

By paying particular attention to the preceding EXPLANATIONS, you will be able perfectly to understand the nature of Decimal Fractions, and clearly to perceive wherein they differ from Vulgar Fractions, and also from WHOLE NUMBERS or INTEGERS.

You will remember, that you learned, in the notation and numeration of figures, or WHOLE NUMBERS, to count from the right hand to the left; and also that they increase in a tenfold proportion from the right to the left; but in the notation and numeration of Decimal Fractions, you must learn to count them from the left to the right, also that they decrease in a tenfold proportion from the left to the right.

NUMERATION TABLE

Of Decimal Fractions.

	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.	Hundred Thousandths.	Millionths.	
$\frac{5}{10}=$,	5						read 5 Tenths.
$\frac{5}{100}=$,	0	5					5 Hundredths.
$\frac{25}{1000}=$,	0	2	5				25 Thousandths.
$\frac{5725}{10000}=$,	5	7	2	5			5725 Ten Thousandths.
$\frac{4}{100000}=$,	0	0	0	0	4		4 Hundred Thousandths.
$\frac{6}{1000}=$,	0	0	6				6 Thousandths
$\frac{1}{1000000}=$,	0	0	0	0	0	1	1 Millionth.

Write the following Sums in Figures.

1. Seventeen, and four tenths.
2. Twenty-five, and twenty-five hundredths.
3. Six, and five thousandths.
4. Four, and one millionth.
5. Seventy-five, and twenty-three hundredths.
6. Forty, and eight thousandths.
7. Sixty-nine, and five tenths.
8. Nine millionths.
9. Twelve, and six hundredths.
10. Thirty, and sixteen hundredths.
11. Ninety-four, and three tenths.
12. Fifty-seven, and two tenths.

ADDITION OF DECIMALS.

RULE.

Q. How do you add decimals?

A. The given numbers must be placed according to their local value, or the value of their places, whether mixed, or pure decimals, so that tenths will stand directly under tenths, hundredths under hundredths, thousandths under thousandths, &c. The numbers must then be added the same as whole numbers, and as many places must be pointed off for decimals, at the right hand, as shall equal the greatest number of decimal places in any of the given numbers.

EXAMPLES

For Theoretical Exercise on a Slate.

1. Add 4 tenths 6 tenths, and 8 tenths together. *Ans.* 1,8.

EXPLANATIONS.

In this example, you add the 8, 6, and 4 tenths together, which make 18 tenths, or 1 unit and 8 tenths, because ten tenths are equal to one unit, or one; and, therefore, you point off below the given points, for when the tenths exceed nine tenths they equal a unit; and the number of figures must increase, and this increase must be units. The reason why you add the same as in whole numbers is very evident, because as the parts diminish in a tenfold proportion from the left hand to the right, so they must increase in a tenfold proportion from the right hand to the left.

,4
,6
,8
1 ,8 *Ans.*

2. Add 1 ,4, 4 ,25, 7 ,45, 2 ,605, ,50 ,05. *Ans.* 16 ,255.

EXPLANATIONS.

You will remember, that the first thing to be done is to place the whole numbers under each other, as in Addition of whole numbers, and the tenths under tenths, hundredths under hundredths, &c. You must begin with the decimal at the right hand to add; carry one for every ten as in whole numbers, and in the final product you must point off as many figures at the right hand for decimals as shall equal the highest number in any of the given numbers to be added.

1 ,4
4 ,25
7 ,45
2 ,605
,50
,05
16 ,255

3. Add six tenths, ninety-nine thousandths, thirty-seven hundredths, nine hundred and five thousandths, and twenty-six thousandths. *Ans.* 2.

4. Add five thousandths, four hundredths, forty-five thousanaths, and four tenths. *Ans.* ,49.

5. Add four tenths, seven hundred and forty-five thousandths, thirty-four ten thousandths, and fifty-six hundredths. *Ans.* 1 ,7084.

6. Add 71 ,467, 27 ,94, 16 ,084, 98 ,009, 86 ,5. *Ans.* 300.

7. Add 19 ,041, 105 ,7, 648 ,006, 19 ,4, 1119 ,05. *Ans.* 1911 ,197.

8. Add $10 ,09, $96 ,75, $144 ,17, $695 .832, $650 ,253. *Ans.* $1597 ,095.

(9.)	(10.)	(11.)	(12.)
,14	183 ,13	5 ,421	,76
,764	91 ,40	2 ,437	,651
,543	942 ,15	8 ,630	5 ,64
,936	507 ,0005	9 ,0005	8 ,02
,834	160 ,005	3 ,5001	2 ,05
3 ,217			

SUBSTRACTION OF DECIMALS.

RULE.

Q. How do you substract decimals?

A. The given numbers must be placed the same as in Addition of decimals, with the less number under the greater, and substracted as whole numbers. Point off the decimals in the difference, or answer, as directed in Addition.

EXAMPLES

For Theoretical Exercise on a Slate.

1. From 75 hundredths take 25 hundredths. *Ans.* ,50.

EXPLANATIONS.

You must first place the less number under the greater, with tenths under tenths, hundredths under hundredths, &c. Begin at the right hand figure, and substract as in whole numbers. As decimals decrease in a tenfold proportion, you will at once see the propriety of borrowing ten and adding it to the upper figure, and of carrying one to the next figure, as in whole numbers, when the figure in the lower line is larger than the upper. After you have substracted the lower line from the

,75
,25
,50 *Ans.*

upper, as in whole numbers, you must, if there be units, or whole numbers, in the given sum, point off as many decimals in the difference, or answer, as the highest number of decimals in either the minuend, or subtrahend, of the given sum.

2. From one and six tenths take four tenths. *Ans.* 1 and 2 tenths.

3. From nine tenths take twenty-five hundredths. *Ans.* ,65.

4. From one take one millionth. *Ans.* ,999999.

5. From forty-five and six tenths take thirty-six and twenty-five hundredths. *Ans.* 9 ,35.

6. From 5 dollars take 6 mills. *Ans.* $4 ,99 ,4.

7. From 6 tenths of a gallon take 35 hundredths of a gallon. *Ans.* ,25 of a gallon.

8. From 5 yards take 25 hundredths of a yard. *Ans.* 4 ,75 yards.

(9.)	(10.)	(11.)	(12.)
456 ,5	148 ,101	29 ,9	27 ,15
67 ,25	84 ,509	17 ,125	1 ,51679

MULTIPLICATION OF DECIMALS.

RULE.

Q. How do you multiply decimals?

A. They must be multiplied the same as whole numbers, and as many figures must be pointed off from the right hand of the product, as there are decimal places in the multiplicand and multiplier. When there are not figures enough in the product, ciphers must be prefixed to the left hand of the product to supply the deficiency.

EXAMPLES

For Theoretical Exercise on a Slate.

1. Multiply 20 ,4 by ,4. *Ans.* 8 ,16.

EXPLANATIONS.

20 ,4
,4
8 ,16

In this example, you multiply by 4, and the product is 816, and you must point off two figures at the right hand, the 16, for decimals, because there are two decimal figures in the multiplicand and multiplier; and the product will be 8 units, or whole numbers, and ,16 hundredths. The reason of your pointing off is very obvious, for the multiplier, 4 tenths, is only one tenth part the value it would be if it stood in the place of units; and, therefore, the product can have only one tenth part the value it would have if it were multiplied by four units; and by pointing off two figures from the right of the product, you give it one tenth the value. Thus, if it were not pointed, the product would be 816; but when pointed, it is only 8 units, and 16 *hun.*, which 8 is, as you will readily perceive, one tenth part of 80.

2. Multiply 55 by ,5. *Ans.* 27 ,5.
3. Multiply ,004 by ,003. *Ans.* ,000012.
4. Multiply 7 ,063 by 4 ,35. *Ans.* 30 ,72405.
5. Multiply 25 ,238 by 12 ,17. *Ans.* 307 ,14646.
6. What will 10 ,5 pounds of butter cost at 25 cents a pound? *Ans.* \$2 ,625, or \$2 ,62 cents, 5 mills.
7. Multiply 18 ,6 by one thousandth. *Ans.* ,0186.
8. Multiply 2461 by ,0529. *Ans.* 130 ,1869.

(9.)	(10.)	(11.)	(12.)
2 ,635	,0372	28 ,33	183 ,5
,025	,0028	4 ,56	126 ,7

(13.)	(14.)	(15.)	(16.)
1 ,526	2 ,639	96 ,023	183 ,051
,036	,536	1 ,912	,002

RULE.—To multiply by 10, 100, 1000, &c., remove the separatrix as many places to the right hand as the multiplier has ciphers. Thus, ,125 multiplied by 10, it would be 1 ,25; multiplied by 100 it would be 12 ,5, and so on.

DIVISION OF DECIMALS.

RULE.

Q. How do you divide decimals?

A. They must be divided as whole numbers, and as many figures must be pointed off from the right hand of the quotient for decimals as the decimal places in the dividend exceed those in the divisor. When the decimal places in the quotient are not enough, ciphers must be prefixed to the left hand of the quotient to supply the deficiency. When the divisor has more decimal places than the dividend, ciphers must be annexed to the right hand of the dividend to supply the deficiency. If there be a remainder, after all the figures of the dividend are brought down, ciphers must be annexed to the remainders until the quotient contains two or three decimal places, or you may carry on the quotient to any degree of exactness; three figures, however, is generally sufficient.

EXAMPLES

For Theoretical Exercise on a Slate.

1. Divide 20 ,5 by ,5 tenths. *Ans.* 41.

EXPLANATIONS.

```
,5)20 ,5
   ----
     41
```

In this example, you divide by the ,5, and the quotient is 41. You will readily see, that the quotient must be double the dividend, for the dividend must contain a half, or five tenths, twice as often as a whole; consequently, the quotient shows that one half a unit, or five tenths, is contained in the dividend, or can be sub-

stracted from the dividend forty-one times. It is very plain, that the quotient would have been 4 and 1 tenth, if the divisor had been 5 units; but the divisor, being only one tenth part of 5 units, is contained in the dividend ten times oftener than 5 units, and the quotient, by pointing off agreeably to the rule, has ten times the value it would have if it were divided by 5 units, as it is now 41.

2. Divide 476 by ,85. *Ans.* 560.

EXPLANATIONS.

In this example, the dividend is composed of whole numbers, and the divisor is a decimal, therefore, you must annex two ciphers, as many as there are decimals in the divisor, at the right hand of the dividend, and the quotient will be whole numbers.

,85)476,00(560

3. Divide 44 by 2 tenths. *Ans.* 220.
4. Divide 463 ,75 by 36 ,4. *Ans.* 1274 ,03.
5. Divide 1 ,28 by 8 ,31. *Ans.* ,134.
6. Divide 10 by 20. *Ans.* ,5.
7. Divide ,875 by 7. *Ans.* ,125.
8. Divide 29 by ,8. *Ans.* 36 ,25.

(9.)	(10.)	(11.)
38)243 ,2	35)3 ,2095	2 ,46)206 ,79

RULE.—To divide by 10, 100, 1000, &c., place the decimal point in the dividend as many places toward the left hand as there are ciphers in the divisor. Thus, 125 divided by 10, it would be 12 ,5; divided by 100, it would be 1 ,25, and so on.

REDUCTION OF DECIMALS.

RULE.

Q. How do you reduce a Vulgar Fraction to its equivalent decimal?

A. Ciphers must be annexed to the numerator, and the numerator must then be divided by the denominator, and the quotient will be the answer required.

EXAMPLES

For Theoretical Exercise on a Slate.

1. Reduce $\frac{1}{2}$ to a decimal. *Ans.* ,5.

EXPLANATIONS.

In this example, you annex a cipher to the 1, the numerator, and divide by the 2, the denominator, and the quotient is ,5. You will perceive, that the value of the fraction is not changed, for 5 tenths bear the same proportion to a unit, or whole number, that the numerator, 1, does to the denominator, 2.

2)1,0
,5

2. Reduce $\frac{3}{4}$ to a decimal. *Ans.* ,75.
3. Reduce $\frac{1}{8}$ to a decimal. *Ans.* ,125
4. Reduce $\frac{1}{5}$ to a decimal. *Ans.* ,2.
5. Reduce $\frac{1}{4}$ to a decimal. *Ans.* ,25.
6. Reduce $\frac{11}{16}$ to a decimal. *Ans.* ,6875.
7. Reduce $\frac{5}{8}$ to a decimal. *Ans.* ,625.
8. Reduce $\frac{17}{20}$ to a decimal. *Ans.* ,85.
9. Reduce $\frac{12}{480}$ to a decimal. *Ans.* ,025.

RULE.

Q. How do you find the value of any given decimal in the terms of an integer, or of the inferiour denominations?

A. The given decimal must be multiplied by the number that it takes of the next inferiour denomination to make a unit, or one, of the denomination of the given decimal, and then as many places must be cut off at the right hand, for a remainder, as there

are decimal places in the given sum, or decimal. The remainder must be multiplied by the parts in the next lower denomination, and cut off, as before; and so through all the parts of the integer; and the several denominations standing on the left hand will be the answer.

EXAMPLES

For Theoretical Exercise on a Slate.

1. What is the value of ,375 of a pound? *Ans.* 7*s.* 6*d.*

EXPLANATIONS.

In this example, you have a fraction of a pound for the given sum. You must first multiply by 20, as twenty shillings make one pound, and shilling is the next lower denomination; and you cut off ,500, the remainder, and have 7*s.* on the left; and then multiply the 500 by 12, and cut off, as before, you find 6*d.* on the left, making 7*s.* 6*d.*, which is the value of £,375, in terms of the inferiour denominations.

```
  ,375
    20
------
 7,500
    12
------
 6,000
```

2. What is the value of ,725 of a day? *Ans.* 17*h.* 24*min.*
3. What is the value of ,3 of a year? *Ans.* 109*d.* 12*h.*
4. What is the value of ,625 of a hundred-weight? *Ans.* 2*qr.* 14*lb.*
5. What is the value of ,7 of a pound troy? *Ans.* 8*oz.* 8*pwt.*
6. What is the value of ,75 of a bushel? *Ans.* 3*p.*
7. What is the value of ,875 of a hogshead? *Ans.* 55*gal.* 0*qt.* 1*pt.*
8. What is the value of ,125 of a gallon? *Ans.* 1*pt.*
9. What is the value of ,75 of a foot? *Ans.* 9*in.*
10. What is the value of ,67 of a league? *Ans.* 2*m.* 0*fur.* 3*po.* 1*yd.* 0*ft.* 3,6*in.*
11. What is the value of ,3375 of an acre? *Ans.* 1*r.* 14*po.*
12. What is the value of ,6875 of a yard? *Ans.* 2*qr.* 3*na.*

RULE.

Q. How do you reduce a given sum, or inferiour denomination, to the decimal of any higher, or superiour denomination?

A. The given sum must be reduced to the lowest denomination mentioned in it, and the proposed integer must be reduced to the same denomination; and the given sum must be divided by the proposed integer, and the quotient will be the decimal required.

EXAMPLES

For Theoretical Exercise on a Slate.

1. Reduce 15*s*. 9*d*. 3*qr*. to the decimal of a pound? *Ans.* ,790625.

EXPLANATIONS.

In this example, you must first reduce 15*s*. 9*d*. 3*qr*. to farthings, as farthings are the lowest denomination mentioned in the given sum. You must then reduce a pound to farthings, the same denomination to which the given sum is reduced, and then divide the given sum, 759, the number of farthings in 15*s*. 9*d*. 3*qr*., by 960, the number of farthings in a pound. As the given sum, 759,

£	*s*. *d*. *qr*.
1	15 9 3
20	12
—	—
20	189
12	4
—	—
240	960)759,0(,790625 the decimal required.
4	6720
—	—
960*qr*.	8700
	8640
	—
	6000
	5760
	—
	2400
	1920
	—
	4800
	4800
	—

is smaller than the proposed integer, 960, you must annex ciphers at the right hand of the given sum. In this manner must any given sum be reduced to the decimal of a higher, or superiour denomination.

2. Reduce 10*s*. 6*d*. to the decimal of a pound. *Ans*. ,525.
3. Reduce 109*d*. 12*h*. to the decimal of a year. *Ans*. ,3.
4. Reduce 4½ calendar months to the decimal of a year. *Ans*. ,375.
5. Reduce 12 drachms to the decimal of a pound avoirdupois. *Ans*. ,046875.
6. Reduce 2*qr*. 14*lb*. to the decimal of a hundred-weight. *Ans*.,625.
7. Reduce 7*oz*. 19*pwt*. to the decimal of a pound troy. *Ans*. 6625.
8. Reduce 3*p*. to the decimal of a bushel. *Ans*. ,75.
9. Reduce 1*qt*. 1*pt*. to the decimal of a gallon. *Ans*. ,375.
10. Reduce 5*fur*. 16*po*. to the decimal of a mile. *Ans*. ,675.
11. Reduce 4*po*. to the decimal of an acre. *Ans*. ,025.
12. Reduce 3*qr*. 2*na*. to the decimal of a yard. *Ans*. ,875.

EXPLANATIONS.

You have now learned all of Arithmetick, that is, you have learned all of the different operations of WORKING FIGURES. You have learned notation and numeration, both of whole numbers and Decimal Fractions. You have learned to add, substract, multiply, and divide numbers, both simple and compound; and you have also learned to add, substract, multiply, and reduce fractions, or parts of whole numbers. All that you now have to become acquainted with, is the different and various applications of the preceding rules, or operations, in the transactions of the various kinds of mechanical and commercial business. All of these operations are performed either by Addition, Substraction, Multiplication, or Division; in some, you must add, substract, and divide; and in others, you must add, substract, multiply, and divide; and, therefore, as the different fundamental rules are used in the operation, these rules have, for the sake of distinction and convenience in reference, names applicable, or appropriate to the application of them in a particular kind of business, or mechanical or commercial transaction. Thus, we call the operation of Multi-

plication and Division, in a certain manner, Interest, Commission, Ensurance, &c. The operation of Multiplication and Division, in another certain manner, Rule of Three, Discount, Barter, Loss and Gain, Tare and Tret, &c. By a very trifling difference in the operation, we have other rules which we call Square and Cube Roots, Position, Arithmetical Progression, &c. &c. But you must not apprehend any difficulty, or be in the least alarmed at this array of new names or rules, for there is no new *principle* to be learned: you have merely to observe the different manner of applying the rules, the *principles* of which you already know, to the various and useful transactions of business, in the different mechanical and commercial pursuits. Only bear this in mind, and all your anticipated difficulties, with regard to the working of new sums, or rules, will vanish.

REDUCTION.

Q. What is Reduction?

A. Reduction teaches to change numbers from one denomination to another, without altering their value.

Q. How many kinds of Reduction are there?

A. Two; Reduction Descending, and Reduction Ascending.

REDUCTION DESCENDING.

Q. What is Reduction Descending

A. Reduction Descending teaches to change, or bring higher denominations into lower; as, pounds into shillings, shillings into pence; pounds into ounces; yards into quarters, &c.

RULE.

Q. How do you reduce high denominations to lower?

A. The number in the highest denomination of the given sum must first be multiplied by that number which it takes of the next lower to make one in that higher, and the figures of the next lower denomination of the given sum must be added in. In this manner must each denomination be multiplied throughout the different denominations; that is, each denomination must be multiplied by that number which it takes of the next lower to make one of that which you are multiplying, always remembering to add in all of the next lower denomination in the given sum when each denominator is multiplied.

REDUCTION ASCENDING.

Q. What is Reduction Ascending?

A. Reduction Ascending teaches to change, or bring lower denominations into higher; as, shillings into pounds, pence into shillings; ounces into pounds; quarters into yards, &c.

RULE.

Q. How do you change low denominations to higher?

A. The lowest denomination given must be divided by that number which it takes of that denomination to make one of the next higher; and in this manner must each denomination be divided up to the denomination required.

Reduction Ascending is precisely the reverse of

Reduction Descending; and, therefore, the different sums in each may be worked reciprocally, as they prove each other.

EXAMPLES

For Theoretical Exercise on a Slate.

FEDERAL MONEY.

1. In \$4, how many cents? *Ans.* 400*c.*

EXPLANATIONS.

In this example, you multiply the \$4 by 100, because 100 cents make a dollar. All that is necessary, however, in reducing federal money, is to add two ciphers to the dollars to reduce them to cents, and three ciphers to reduce them to mills;	\$ 4 100 —— 400*c.* *Ans.*

or, if the sum consist of dollars and cents, add one cipher to reduce them to mills, which is the same as multiplying by 10, 100, 1000, as you remember in the multiplication of whole numbers, by 10, 100, 1000, &c. When the given sum is composed of dollars, cents, and mills, you have only to remove the comma, or separatrix, and the answer will be in mills. To bring mills into cents, you must cut off one figure at the right hand, by the separatrix, and the figures left of the separatrix will be cents; and to bring mills into dollars and cents, you must cut off one figure for mills, and two more for cents, and the figures at the left of the separatrix will be dollars.

2. In 400 cents, how many dollars? *Ans.* \$4.
3. In \$8, how many mills? *Ans.* 8000*m.*
4. In 8000 mills, how many dollars? *Ans.* \$8.
5. In \$800 and 1 mill, how many mills? *Ans.* 800001*m.*
6. In 800001 mills, how many dollars? *Ans.* 800,00,1*m.*
7. In \$1, 11 cents, and 1 mill, how many mills? *Ans.* 1111*m.*
8. In 1111 mills, how many dollars? *Ans.* \$1,11,1*m.*

STERLING OR ENGLISH MONEY.

1. In £31 11*s.* 10*d.* 1*qr.*, how many farthings? *Ans.* 30329*qr.*

EXPLANATIONS.

£	s.	d.	qr.
31	11	10	1

20 shillings in a pound.

631 shillings.
12 pence in a shilling.

7582 pence.
4 farthings in a penny.

30329 farthings.

In this example, you must multiply the £31 by 20, because twenty shillings make a pound, and add in the 11*s.*, in the given sum, with the product of shillings. You must multiply the 631 shillings by 12, because twelve pence make a shilling, and add in the 10*d.*, in the given sum, with the product of pence. You must multiply the 7582 pence by 4, because four farthings make a penny, and add in the 1*qr.*, in the given sum, with the product of farthings. In this easy manner are all large denominations changed into smaller, by multiplying the given sum by that number which it takes of the next lower to make one of the denomination you are multiplying, always remembering to add in the lower denomination of the given sum, in the product of that of the same denomination; as, in multiplying hours, you multiply by 60, because sixty minutes make an hour, &c.

2. In 30329 farthings, how many pounds? *Ans.* £31 11*s.* 10*d.* 1*qr.*

EXPLANATIONS.

qr.
4)30329

12)7582 1*qr.*

2 | 0(63 | 1 10*d.*

£31 11*s.*

In this example, you must first divide by 4, because four farthings make one penny, the next higher denomination, and the remainder will be farthings. Divide the 7582 pence by 12, because twelve pence make a shilling, the next higher denomination, and the remainder will be pence. Divide 631 shillings by 20, because twenty shillings make a pound, the next higher denomination, and the remainder will be shillings, and you will then have the answer, £31 11*s.* 10*d.* 1*qr.* In this manner are all small denominations changed into larger, by dividing the given sum by that number which it takes of that to make one of the next

higher denomination; as, in dividing seconds, or minutes, you divide by 60; in dividing drachms, or ounces avoirdupois, you divide by 16, because sixteen drachms make an ounce, &c.

3. In £18 12*s.* 7*d.*, how many pence? *Ans.* 4471*d.*

4. In 4471 pence, how many pounds? *Ans.* £18 12*s.* 7*d.*

5. In £61 12*s.*, how many shillings? *Ans.* 1232*s.*

6. In 1232 shillings, how many pounds? *Ans.* £61 12*s.*

7. In 41 guineas, at 28 shillings each, how many pence? *Ans.* 13776*d.*

8. In 13776 pence, how many guineas at 28 shillings each? *Ans.* 41 *guineas.*

9. In 320 pistoles, at 22 shillings each, how many shillings? *Ans.* 7040*s.*

10. In 7040 shillings, how many pistoles? *Ans.* 320 *pistoles.*

11. In 24 dollars, at 8 shillings each, how many pence and shillings? *Ans.* 192*s.* 2304*d.*

12. In 2304 pence, how many dollars, at 8 shillings each? *Ans.* $24.

13. In 48 moidores, at 36 shillings each, how many shillings? *Ans.* 1728*s.*

14. In 1728 shillings, how many moidores, at 36 shillings each? *Ans.* 48 *moidores.*

TIME.

1. In 287 days, how many seconds? *Ans.* 24796800*sec.*

2. In 24796800 seconds, how many minutes, hours, and days? *Ans.* 413280*min.* 6888*h.* 287*d.*

3. In 25 weeks, how many days? *Ans.* 175*d.*

4. In 175 days, how many weeks? *Ans.* 25*w.*

5. In 30 years, how many seconds, allowing 365 days and 6 hours to the year? *Ans.* 946728000*sec.*

6. In 946728000 seconds, how many years? *Ans.* 30*y.*

AVOIRDUPOIS WEIGHT.

1. In 20 ounces, how many drachms? *Ans.* 320*dr.*

2. In 320 drachms, how many ounces? *Ans.* 20*oz.*

3. In 1 tun, how many drachms? *Ans.* 573440*dr.*

4. In 573440 drachms, how many tuns? *Ans.* 1*T.*

5. In 45*T.* 14*cwt.* 3*qr.* 21*lb.* 13*oz.* 7*dr.*, how many drachms? *Ans.* 26233308*dr.*

6. In 26233303 drachms, how many tuns? *Ans.* 45*T.* 14*cwt.* 3*qr.* 21*lb.* 13*oz.* 7*dr.*

APOTHECARIES WEIGHT.

1. In 14 pounds, how many grains? *Ans.* 80640*gr.*
2. In 80640 grains, how many pounds? *Ans.* 14℔.
3. In 8 ounces, how many scruples? *Ans.* 192℈.
4. In 192 scruples, how many ounces? *Ans.* 8℥.
5. In 8℔ 6℥ 4ʒ 1℈ 12*gr.*, how many grains? *Ans.* 49232*gr.*
6. In 55799 grains, how many pounds? *Ans.* 9℔ 8℥ 1ʒ 2℈ 19*gr.*

TROY WEIGHT.

1. In 25 pounds, how many grains? *Ans.* 144000*gr.*
2. In 144000 grains, how many pounds? *Ans.* 25*lb.*
3. In 8 pounds, how many penny-weights? *Ans.* 1920*pwt.*
4. In 1920 penny-weights, how many pounds? *Ans.* 8*lb.*
5. In 16*lb.* 10*oz.* 18*pwt.* 5*gr.*, how many grains? *Ans.* 97397*gr.*
6. In 97397 grains, how many pounds? *Ans.* 16*lb.* 10*oz.* 18*pwt.* 5*gr.*

DRY MEASURE.

1. In 8 bushels, how many quarts? *Ans.* 256*qt.*
2. In 256 quarts, how many bushels? *Ans.* 8*bu.*
3. In 80 bushels, how many pints? *Ans.* 5120*pt.*
4. In 5120 pints, how many bushels? *Ans.* 80*bu.*
5. In 46*bu.* 3*p.* 7*qt.*, how many quarts? *Ans.* 1503*qt.*
6. In 1503 quarts, how many bushels? *Ans.* 46*bu.* 3*p.* 7*qt.*

WINE MEASURE.

1. In 35 gallons, how many pints? *Ans.* 280*pt.*
2. In 280 pints, how many gallons? *Ans.* 35*gal.*
3. In 9 tuns, how many quarts? *Ans.* 9072*qt.*
4. In 9072 quarts, how many tuns? *Ans.* 9*T.*
5. In 116 barrels, how many gills? *Ans.* 116928*gi.*
6. In 116928 gills, how many pints, quarts, barrels? *Ans.* 29232*pt.* 14616*qt.* 116*bar.*

LONG MEASURE.

1. In 49 feet, how many inches? *Ans.* 588*in.*
2. In 588 inches, how many feet? *Ans.* 49*ft.*
3. In 49 yards, how many barley-corns? *Ans.* 5292*bc.*
4. In 10*m.* 4*fur.* 24*po.* 10*ft.* 4*in.*, how many inches? *Ans.* 670156*in.*
5. In 40 miles, how many yards? *Ans.* 70400*yd.*
6. In 70400 yards, how many miles? *Ans.* 40*m.*

LAND OR SQUARE MEASURE.

1. In 6 square feet, how many square inches? *Ans.* 864*in.*
2. In 864 square inches, how many square feet? *Ans.* 6*S. F.*
3. In 150 acres, how many square poles? *Ans.* 24000*po.*
4. In 24000 square poles, how many acres? *Ans.* 150*a.*
5. In 241*a.* 3*r.* 25*po.*, how many square poles? *Ans.* 38705*po.*
6. In 38705 square poles, how many acres? *Ans.* 241*a.* 3*r.* 25*po.*

SOLID OR CUBICK MEASURE.

1. In 42 cords of wood, how many solid feet? *Ans.* 5376*ft.*
2. In 5376 solid feet, how many cords? *Ans.* 42*C.*
3. In 18 tuns of hewn timber, how many solid feet and inches? *Ans.* 900*ft.* 1555200*in.*
4. In 900 solid feet, how many tuns of hewn timber? *Ans.* 18*T.*
5. In 36 cords, how many solid feet? *Ans.* 4608*ft.*
6. In 4608 solid feet, how many cords? *Ans.* 36*C.*

CLOTH MEASURE.

1. In 44 yards, how many quarters? *Ans.* 176*qr.*
2. In 176 quarters, how many yards? *Ans.* 44*yd.*
3. In 190 yards, how many nails? *Ans.* 3040*na.*
4. In 3040 nails, how many yards? *Ans.* 190*yd.*
5. In 122 ells English, how many quarters? *Ans.* 610*qr.*
6. In 3840 nails, how many yards, ells English, and ells Flemish? *Ans.* 240*yd.* 192*E. E.* 320*E. F.*

CIRCULAR MOTION.

1. In 11 signs, how many degrees? *Ans.* 330°
2. In 330 degrees, how many signs? *Ans.* 11*S.*
3. In 4*S.* 3° 18′ 27″, how many seconds? *Ans.* 443907″.
4. In 443907 seconds, how many signs? *Ans.* 4*S.* 3° 18′ 27″.
5. In 360 degrees, how many seconds? *Ans.* 1296000″.
6. In 1296000 seconds, how many degrees? *Ans.* 360°.

PAPER.

1. In 20 bundles, how many reams? *Ans.* 40*r.*
2. In 40 reams, how many bundles? *Ans.* 20*bu.*
3. In 120 reams, how many quires? *Ans.* 2400*q.*
4. In 2400 quires, how many reams? *Ans.* 120*r.*
5. In 480 quires, how many sheets? *Ans.* 11520*s.*
6. In 11520 sheets, how many quires? *Ans.* 480*q.*

REDUCTION OF CURRENCIES.

Q. What is Reduction of Currencies?

A. Reduction of Currencies teaches to find the value of the coin, or currency, of one state, or country, in that of another. The same denominations and coin are generally used in the different states and countries, but the standard value frequently differs in each. Thus, a dollar is reckoned, in New York, Ohio, and North Carolina, 8*s.*, called *New York* currency.

In New England, Virginia, Kentucky, and Tennessee, 6*s.*, called *New England* currency.

In New Jersey, Pennsylvania, Delaware, and Maryland, 7*s.* 6*d.*, called *Pennsylvania* currency.

In South Carolina and Georgia, 4*s.* 8*d.*, called *Georgia* currency.

In Canada and Nova Scotia, 5*s.*, called *Canada*, or *Halifax* currency.

In England, 4*s.* 6*d.*, called *English* or *Sterling* currency.

RULE.

Q. How do you reduce the currency of each state, or country, to federal money?

A. The given sum must first be reduced to shillings, sixpences, or to pence, and then divided by the number of shillings, sixpences, or pence, in a dollar, as it is reckoned in each state.

EXAMPLES.

1. Reduce £192, New York currency, to federal money. *Ans.* \$480.

EXPLANATIONS.

In this example, you multiply the given sum, £192, by 20, which reduces it to shillings, and then divide the product by 8, the number of shillings in a dollar New York currency, and the answer will be dollars.

```
   £
  192
   20 shillings in a pound.
 ----
8)3840
 ----
 $480
```

2. Reduce £28 11*s.* 6*d*, New England currency, to federal money. *Ans.* \$95,25.

EXPLANATIONS.

In this example, you must first reduce the £28 11*s.* 6*d.* to pence, and then divide the product by 72, the number of pence in a dollar New England currency, and the answer will be dollars. When there is a remainder, add ciphers, and divide as before, the answer will be decimals of a dollar, or cents and mills.

```
    £  s. d.
    28 11 6
    20
   ---
   571
    12
   ---    $ c.
72)6858(95,25
   648
   ---
    378
    360
    ---
     180
     144
     ---
      360
      360
```

3. Reduce £206 New York currency to federal money. *Ans.* $515.

4. Reduce £241 New England currency to federal money. *Ans.* $803,33*c.*

5. Reduce £240 Pennsylvania currency to federal money. *Ans.* $640.

6. Reduce £250 10*s.* Pennsylvania currency to federal money. *Ans.* $668.

7. Reduce £417 14*s.* 6*d.* Georgia currency to federal money. *Ans.* $1790,25*c.*

8. Reduce £147 12*s.* 8*d.* Georgia currency to federal money. *Ans.* $632,71,4*m.*

9. Reduce £45 English or sterling money to federal money. *Ans.* $200.

10. Reduce £10 18*s.* 0*d.* 2*qr.* English or sterling money to federal money. *Ans.* $48,45,2*m.*

11. Reduce £241 18*s.* 9*d.* Canada currency to federal money. *Ans.* $967,75*c.*

12. Reduce £50 Canada currency to federal money. *Ans.* $200.

RULE.

Q. How do you reduce federal money to the currency of each state?

A. The given sum in cents must be multiplied by the number of pence in a dollar, and two figures must be cut off at the right of the product, and those at the left will be the answer in pence. If the two figures cut off at the right hand (not ciphers) be multiplied by 4, and two figures again cut off, those at the left will be farthings.

EXAMPLES.

1. Reduce $271,50 cents to New York currency. *Ans.* £108 12*s.*

EXPLANATIONS.

In this example, you multiply the dollars and cents by 96, because 96 pence make one dollar, in New York currency; and, therefore, you will readily perceive, that every hundred cents, in New York currency, must be diminished four in number to reduce them to pence, as 96 pence are equal to 100 cents; and you will also perceive, that to multiply by 96, and cut off two of the right hand figures, will lessen the given number four on each one hundred; for cutting off two figures, is dividing the product by 100.

```
     271,50
         96
   --------
     162900
    244350
   --------
 12)26064,00
   --------
 2|0)217|2
   --------
   £108 12s.
```

2. Reduce $410 to New York currency. *Ans.* £164.
3. Reduce $196 to New England currency. *Ans.* £58 16s.
4. Reduce $1368 to New England currency. *Ans.* £410 8s.
5. Reduce $1402,68c. to Pennsylvania currency. *Ans.* £526 0s. 1d.
6. Reduce 15 cents to Pennsylvania currency. *Ans.* 1s. 1d. 2qr.
7. Reduce $1050,80c, to Georgia currency. *Ans.* £245 3s. 8d. 3qr.
8. Reduce $3837 to Canada currency. *Ans.* £959 5s.

TABLES OF COINS.

A Table of Foreign Coins, &c., with their value in Federal Money, as established by an act of Congress.

	$ c. m.		$ c. m.
Pound Sterling, - -	4, 44, 0	The Guilder of the United Netherlands, -	0, 39, 0
Pound of Ireland, - -	4, 10, 0	Mark Banco of Hamburgh, - - - - -	0, 33, 5
Pagoda of India, - -	1, 94, 0	Livre Turnois of France, - - - -	0, 18, 5
Tale of China, - -	1, 48, 0	Real Plate of Spain, -	0, 10, 0
Mill-rea of Portugal, -	1, 24, 0		
Ruble of Russia, - -	0, 66, 0		
Rupee of Bengal, - -	0, 55, 5		

A TABLE OF COINS,

Which pass Current in the United States of America, with their Sterling and Federal value.

Names of Coins.	Standard Weight.		Sterling Money of Great Britain.			N. Hampshire, Massachusetts, Rhode Island, Connecticut, and Virginia.			N. York, Ohio, and North Carolina.			N. Jersey, Pennsylvania, Delaware, and Maryland.			South Carolina and Georgia.			Fed. Value. Dolls.	Cents.	Mills.
GOLD.	pwt.	gr.	£	s.	d.	£	s.	d.	£	s.	d.	£.	s.	d.	£	s.	d.	D.	C.	M.
A Johannes,	18	0	3	12	0	4	16	0	6	8	0	6	0	0	4	0	0	16,	00,	0
A Half Johannes,	9	0	1	16	0	2	8	0	3	4	0	3	0	0	2	0	0	8,	00,	0
A Doublon,	16	21	3	6	0	4	8	0	5	16	0	5	12	6	3	10	0	14,	93,	3
A Moidore,	6	18	1	7	0	1	16	0	2	8	0	2	5	0	1	8	0	6,	00,	0
An English Guinea,	5	6	1	1	0	1	8	0	1	17	0	1	15	0	1	1	9	4,	66,	7
A French Guinea,	5	5	1	1	0	1	7	6	1	16	0	1	14	6	1	1	3	4,	69,	0
A Spanish Pistole,	4	6	0	16	6	1	2	0	1	9	0	1	8	0	0	18	0	3,	77,	3
A French Pistole,	4	4	0	16	0	1	2	0	1	8	0	1	7	6	0	17	6	3,	66,	7
SILVER.																				
An English or French Crown,	19	0	0	5	0	0	6	8	0	8	9	0	8	3	0	5	0	1,	10,	0
The Dollar of Spain, Sweden, or Denmark,	17	6	0	4	6	0	6	0	9	8	0	0	7	6	0	4	8	1,	00,	0
An English Shilling,	3	18	0	1	0	0	1	4	0	1	9	0	1	8	0	1	0	0,	22,	2
A Pistareen,	3	11	0	0	10½	0	1	2	0	1	7	0	1	6	0	0	11	0,	20,	0

All other Gold Coins of equal fineness, at 89 cents per pwt., and silver at 111 cents per oz.

INTEREST.

Q. What is INTEREST?

A. Interest is a certain premium per cent., or sum paid, or allowed by the borrower to the lender for the use of money lent.

Q. How many kinds of Interest are there?

A. Two; Simple and Compound.

Q. What are the three things to be noticed in Interest?

A. *Principal*, the money or sum lent; *rate*, the sum per cent. agreed on, or established by law; and the *amount*, the principal and interest added together.

EXPLANATIONS.

In most of the states, the rate of interest is limited by law to six per cent., that is, six cents for the use of one hundred cents for the term of one year; or six dollars, or six pounds, for the use of one hundred dollars, or one hundred pounds for one year, and the like proportion for a greater or less sum, or for a longer or shorter time. In the State of New York, the per cent. established by law is seven; in England, five per cent. is the rate established by law. What is termed lawful or legal interest, is that which is allowed by the laws of the state. The lender is not permitted to receive a larger per cent. for the use of his money than is allowed by the laws of the state in which he resides. Interest is one of the most important rules of Arithmetick, and it is one with which every person should be familiarly acquainted.

SIMPLE INTEREST.

Q. What is Simple Interest?

A. Simple Interest is that which is allowed on the principal only.

RULE.

Q. How do you find the interest of any given sum for one year?

A. The principal must be multiplied by the rate per cent., and the product must be divided by one hundred, and the quotient will be the answer; or cut off two figures from the right hand of the product, and those on the left will be the interest of that principal for one year.

EXAMPLES.

1. What is the interest of 300 dollars for one year, at 7 per cent.? *Ans.* $21.

EXPLANATIONS.

The rule of Interest is performed by Multiplication and Division. In this example, you multiply the principal, 300 dollars, by 7 dollars, the interest of one hundred dollars for a year; and, therefore, the product is 100 times too large, consequently, it must be divided by 100 for the interest. But, to divide by 100, you will remember, that you have merely to cut off two figures at the right hand, and the figures on the left will be the answer. In order to render this perfectly plain to you, you must consider that the multiplier is, in fact, seven cents, or seven hundredths of a dollar, instead of seven dollars; and, therefore, you point off two figures from the right hand, agreeably to the rule of multiplying decimals, as will appear obvious from the following operation of the present example. Thus,

$	
300	
7	
$21,00	

,07*c.*	interest of $1 for one year, at 7 per cent.
300	number of dollars in the principal.
$21,00	interest of $300 for one year, at 7 per cent.

2. What is the interest of $400 for one year, at 6 per cent.? *Ans.* $24.

3. What is the interest of \$575 for one year, at 7 per cent.? *Ans.* \$40,25*c.*

4. What is the amount of \$5 for one year, at 6 per cent.? *Ans.* \$5,30*c.*

5. What is the interest of \$25 for one year, at 5 per cent.? *Ans.* \$1,25*c.*

6. What is the interest of \$157 for one year, at 4 per cent.? *Ans.* \$6,28*c.*

7. What is the interest of \$135,25*c.* for one year, at 6 per cent.? *Ans.* \$8,11,5*m.*

8. What is the interest of \$784,65*c.* for one year, at 6 per cent.? *Ans.* \$47,07,9*m.*

9. What is the amount of £200 for one year, at 7 per cent.? *Ans.* £214.

10. What is the interest of £560 for one year, at 5 per cent.? *Ans.* £28.

RULE.

Q. How do you find the interest of any given sum for any number of years, and parts of a year?

A. First find the interest of the given sum for one year, and then multiply the interest of one year by the given number of years, and the product will be the answer for that time. If there be parts of a year, you must take parts of the sum; as, for six months, you must take half of the product for one year; for three months one quarter; for four months one third; for eight months two thirds; for nine months three quarters, &c. For fifteen days you must take one half of the product for one month; for ten days one third; for twenty days two thirds, &c.

EXAMPLES.

1. What is the interest of £129 15*s.* 4*d.* for two years at 6 per cent.? *Ans.* £15 11*s.* 5*d.*

EXPLANATIONS.

In this example, you multiply by the rate per cent., as before, and obtain the interest for one year; then multiply the interest of one year, £7 15*s.* 8*d.* 2*qr.*, by 2, and the product will give the interest for two years.

£ *s.* *d.*
129 15 4
6
7,78 12 0
20
15,72
12
8,64
4
2,56

£ *s.* *d.* *qr.*	
7 15 8 2	*in.* for 1 *yr.*
2	
£15 11 5 0	*in.* for 2 *yr.*

2. What is the interest of \$384,25*c.* for 1 year, 9 months, and 15 days, at 7 per cent.? *Ans.* \$48,19*c.*

EXPLANATIONS.

In this example, you must first multiply by the 7, the rate per cent., and you will find the interest for one year. You must take half the interest for one year for the six months; the half of that product for 3 months; then one sixth of the interest for 3 months for 15 days, as 15 days are one sixth part of 3 months; and you must then add these several products together, and their amount will be the answer.

	\$ *c.*	
	384,25	
	7	
6 *mo.* $\frac{1}{2}$)	26,89,75	*in.* for 1 *yr.*
3 *mo.* $\frac{1}{2}$) of 6 *mo.*	13,44,87	*in.* for 6*mo.*
15 *da.* $\frac{1}{6}$) of 3*mo.*	6,72,43	*in.* for 3*mo.*
	1,12,07	*in.* for 15*da.*
	\$48,19,12*m.*	*Ans.*

3. What is the interest of \$270 for one year and three months, at 5 per cent.? *Ans.* 16,87,5*m.*

4. What is the interest of \$368,84*c.* for three years and six months, at 7 per cent.? *Ans.* \$90,36,5*m.*

5. What is the interest of \$92 for two years, at 7 per cent.? *Ans.* 12,88*c.*

6. What is the interest of \$350 for two years, at 5 per cent.? *Ans.* \$35.

7. What is the interest of $279,87c. for two years and six months, at 7 per cent.? *Ans.* $48,97,7*m*.

8. What is the interest of $768.50 for one year, nine months, and fifteen days, at 7 per cent.? *Ans.* $96.382

9. What is the interest of $600 for three years, at 5 per cent.? *Ans.* $90.

10. What is the interest of $184,47c. for ten months and twenty days, at 6 per cent.? *Ans.* $9,83,8*m*.

RULE.

Q. When there is a fraction in the rate per cent., as, $4\frac{1}{2}$, $7\frac{1}{4}$, &c., how do you find the interest?

A. The principal must be multiplied by the rate per cent., as before; and to the product must be added $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{1}{3}$, &c., of the principal, and the product thus increased must be divided by 100, as in the preceding examples.

EXAMPLES.

1. What is the interest of $300 for one year, at $6\frac{1}{4}$ per cent.? *Ans.* $18,75*c*.

EXPLANATIONS.

In this example, you first multiply by the 6, and then divide the principal, 300, by $\frac{1}{4}$, which is 75, added to the 18, makes the product $18,75.

$$\begin{array}{r} \$ \\ \tfrac{1}{4})300 \\ 6\tfrac{1}{4} \\ \hline 1800 \\ 75 \\ \hline \$18,75 \end{array}$$

2. What is the interest of $842 for two years, at $5\frac{1}{2}$ per cent.? *Ans.* $92,62*c*.

3. What is the interest of $428 for one year, at $6\frac{1}{4}$ per cent.? *Ans.* $26,75*c*.

4. What is the interest of $524 for three years, at $5\frac{1}{4}$ per cent.? *Ans.* $82,53*c*.

5. What is the interest of $500 for five years, at $7\frac{1}{2}$ per cent.? *Ans.* $187,50*c*.

RULE.

Q. How do you find the interest for days, at any rate per cent.?

A. The given principal must be multiplied by the given number of days, and that product must be multiplied by the rate per cent., whether 6, 5, 7, 4, $3\frac{1}{2}$, &c.; thus ,06, ,05, ,07, 04, 03, 5, &c.; and the last product must be divided by 365, (the days in a year,) and it will give the interest in dollars, and parts of a dollar.

EXAMPLES.

1. What is the interest of \$175,58*c.* for 85 days, at 6 per cent.? *Ans.* \$2,45,3$\frac{3}{10}$*m.*

EXPLANATIONS.

In this example, you first multiply the given sum by 85, the given number of days; that product by the rate per cent., ,06, and divide by 365, the number of days in a year.

```
                          $   c.
                        175,58 given principal
                            85 number of days
                        ------
                         87790
                        140464
                        ------
                      14924,30
                           ,06 inter. of a dollar
                     ---------
Days in a year, 365)895,4580(2,45,3, 3 Ans.
                    730
                    ----
                    1654
                    1460
                    ----
                     1945
                     1825
                     ----
                      1208
                      1095
                      ----
                       1130
                       1095
                       ----
                         35
```

2. What is the interest of $689,80c. for forty days, at 3½ per cent.? *Ans.* $2,64,5m.

3. What is the interest of $1172,50c. for forty-two days, at 3 per cent.? *Ans.* $4,04,7m.

4. What is the interest of $1570,56c. for one hundred and ten days, at 5 per cent.? *Ans.* $23,66,5m.

RULE.

Q. How do you compute the interest on a note, bond, or obligation, &c., on which endorsements or partial payments have been made?

A. First cast the interest on the several payments from the time they were paid to the time of settlement, and find their amount; then find the amount of the whole principal for the whole time; and then deduct the amount of the several payments from the amount of the principal.

EXAMPLES.

1. A bond was given May 17, 1823, for $675; interest at 6 per cent., and three payments were endorsed on it as follows:

First payment, $148, June 7, 1824.

Second payment, $341, September 17, 1826.

Third payment, $99, February 2, 1828; how much remained due on the bond, July 17, 1828? *Ans.* $219,52c.

EXPLANATIONS.

You first find the interest on $148, the first payment, to the time of settlement, July 17, 1828, which is 4*yr.* 1$\frac{1}{3}$*mo.*, amounting, with the payment, to $184,50. You then find the interest on the second payment, $341, as before, for 1*yr.* 10*mo.*, which, with the payment, amounts to $378,51; and you then find the interest on the third payment, $99, as before, for 5½*mo.*, which, with the payment, amounts to $101,72. You must then add the several payments, making $664,73; and then find the amount of the whole principal for the whole time, 5*yr.* 2*mo.*, which will be $884,25; from which you must deduct the amount of the whole payments, $664,73, and the remainder due will be $219,52.

$ c.		
148,00	first payment, June 7, 1824.	yr. mo.
36,50	interest to July 17, 1828.	4 $1\frac{1}{3}$
184,50	amount.	
341,00	second payment, September 17, 1826.	yr. mo.
37,51	interest to July 17, 1828.	1 10
378,51	amount.	
99,00	third payment, February 2, 1828.	mo.
2,72	interest to July 17, 1828.	$5\frac{1}{2}$
101,72	amount.	

$\left.\begin{array}{r}184{,}50\\378{,}51\\101{,}72\end{array}\right\}$ several amounts

664,73	total amount of payments.	
		yr. mo.
675,00	bond, dated May 17, 1823.	5 2
209,25	interest to July 17, 1828.	
884,25	amount of the note.	
664,73	amount of payments.	

$219,52c. remained due on the bond, July 17, 1828.

NOTE.—The preceding rule will answer for short periods of time, but when applied to a long course of years, will be found to be very erroneous. Different rules have been established in the different states relative to the computation of interest on notes, bonds, &c.

The following Rule is established for the practice of the Courts in the State of New York.

"The Rule for casting interest, where partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due. If the payment exceeds

the interest, the surplus goes toward discharging the principal, and the subsequent interest is to be computed on the balance of principal remaining due. If the payment be less than the interest, the surplus of interest must not be taken to augment the principal; but interest continues on the former principal until the period when the payments, taken together, exceed the interest due; and then the surplus is to be applied toward discharging the principal, and interest is to be computed on the balance of principal as aforesaid."

The following Rule was established by the Superiour Court of Massachusetts in 1821.

"Compute the interest on the principal sum, from the time when the interest commenced, to the first time when a payment was made, which exceeds, either alone or in conjunction with the preceding payments, if any, the interest at that time due; add that interest to the principal, and from the sum substract the payment made at that time, together with the preceding payments, if any; and the remainder forms a new principal, on which compute interest, as upon the first principal, and from the amount substract the payment; and proceed in this manner to the time of the judgement or settlement."

The following Rule was established by the Superiour Court of Connecticut in 1784.

"Compute the interest to the time of the first payment, if that be one year or more from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after-payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and in like manner, from one payment to another, till all the payments are absorbed; *provided*, the time between one payment and another be one year or more. But if any payment be made before one year's interest hath accrued, then compute the interest on the principal sum due on the obligation for one year, add it to the principal, and compute the interest on the sum paid, from the time it was paid, up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest added as above. If any payment be made of a less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum, for any period."

2. A gave B a bond for $2000, dated April 15, 1822, on which are the following endorsements:

			yr.	mo.	da.
August 20, 1823, $160,50,	-	-	- 1	4	5
September 20, 1825, $190,36,	-	-	- 2	1	0
May 15, 1827, $575,20,	-	-	- 1	7	25
June 25, 1829, $645,75,	-	-	- 2	1	10

What remained due upon the bond October 15, 1831, interest at 7 per cent.? *Ans.* $1630.27

NOTE.—The preceding sum is worked agreeably to the rule established for the practice of the courts in the State of New York.

3. A note was given June 15, 1829, for $1200, on which the following endorsements were made:

				yr.	mo.
June 15, 1830, $1000,	-	-	-	- 1	0
December 15, 1830, $200,	-	-	-	- 0	6

What remained due on the note June 15, 1831, interest at 6 per cent.? *Ans.* $82,56,4*m*.

NOTE.—The rule established by Massachusetts, is, perhaps, for simplicity and correctness, equal, if not superiour to any.

4. A bond was given February 4, 1825, for $1000, interest at 7 per cent., on which the following payments were made:

				yr.	mo.	da.
March 19, 1826, $200,	-	-	-	- 1	1	15
July 29, 1827, $500,	-	-	-	- 1	4	10
December 14, 1827, $260,	-	-		- 0	4	15

What remained due on the bond January 24, 1829? *Ans.* $231,26*c*.

5. A note was given November 1, 1826, for $1000, interest at 7 per cent., on which were the following endorsements:

February 10, 1827, $400,
August 6, 1828, $500,

What remained due on the note July 10, 1829? *Ans.* $195, 81,2*m*.

COMPOUND INTEREST.

Q. What is Compound Interest?

A. Compound Interest is that which arises from the *principal* and *interest*, or interest upon interest; the interest being added to the principal, and continuing in the hands of the borrower, becomes a part of the principal at the end of the stated time of payment.

EXPLANATIONS.

If $100 be on interest, at 7 per cent. per annum, at the end of a year, $107 will be due to the lender; and this $107 will be the principal for the second year to draw interest, and so on; the *principal* and *interest* being added together as the interest becomes due; and it is for this reason that it is called Compound Interest.

RULE.

Q. How do you find the Compound Interest on any given sum for a given time?

A. First find the amount of the given sum for the first year, the same as in Simple Interest, which will be the principal for the second year; then find the amount of this new principal, as before, for the second year, and that will be the principal for the third year, and so on for any number of years; and then substract the given principal from the last amount, and the difference, or remainder, will be the Compound Interest.

EXAMPLES.

1. What is the Compound Interest of $100, for 4 years, at 7 per cent.? *Ans.* $31,07,9*m*.

EXPLANATIONS.

You will readily perceive, that the principle of the rule of working Compound Interest is the same as that of Simple Interest, the only difference is, that in Simple Interest, the interest is reckoned on the sum for the given time, and, in Compound Interest, the interest is reckoned at the end of each year, and added to the principal, and, therefore, becomes a part of the principal upon which interest is reckoned. Thus, \$100, for 1 year, at 7 per cent. per annum, is \$107, which is the principal for the second year; and the amount of \$107 is \$114,49*c.*; this is the principal for the third year, and so on.

\$	
100	principal.
7	per cent.
700	interest for 1 year.
100	
107,00	am't at the end of the 1st *yr.*
7	
7,4900	
107,00	
114,49,00	am't at the end of the 2d *yr.*
7	
801,4300	
11449	
122,50,4300	am't at the end of the 3d *yr*
7	
8,57,530100	
122,50,43	
131,07,960100	am't at the end of the 4th *yr.*
100	principal deducted.
31,07,960100	Com. In. of \$100 for 4*yr.*

2. What is the Compound Interest of \$400, for 5 years, at 6 per cent.? *Ans.* \$135,29*c.*

3. What is the Compound interest of \$760, for 3 years, at 6 per cent.? *Ans.* \$145,17,2*m.*

4. What will \$500 amount to in 3 years, at 7 per cent., Compound Interest? *Ans.* \$612,52*c.*

5. What is the Compound Interest of £400, for 4 years, at 6 per cent.? *Ans.* £104 19*s.* 9¼*d.*

6. What will $2000 amount to in 4 years, at 7 per cent., Compound Interest? *Ans.* $2621,59,2*m*.

7. What is the Compound Interest of $400, for 3½ years, at 6 per cent.? *Ans.* $90,69,8*m*.

ENSURANCE, COMMISSION OR FACTORAGE, BROKERAGE AND BUYING AND SELLING STOCKS.

Q. What are Ensurance, Commission or Factorage, Brokerage, and Buying and Selling Stocks?

A. They are allowances made to Ensurers, Factors, Brokers, and Buyers, at a stipulated rate per cent.

RULE.

Q. How do you find the Ensurance, Commission, or Brokerage, on any given sum?

A. The given sum must be multiplied by the rate per cent., and the product divided by 100, the same as in finding the interest of any given sum for one year only.

ENSURANCE.

NOTE.—Ensurance is a premium, at a certain per cent., allowed to companies and persons, for securing or indemnifying against loss of property, such as houses, ships, merchandise, &c., which may happen from fire, storms, &c. The writing by which the contract of indemnity is binding upon the parties, is called the *policy*. The average loss is 5 per cent., that is, if a person having his property ensured does not suffer damage or loss exceeding 5 per cent., he must bear it himself, and can not call on the *ensurers*.

EXAMPLES.

1. What is the ensurance of $3250, at 3 per cent.? *Ans.* $97,50.

EXPLANATIONS.

In this example, you multiply by 3, the rate per cent., and divide by 100, or, rather, you cut or point off two figures at the right hand, precisely as in obtaining the Simple Interest on any given sum for one year.	\$ 3250 3 —— 97,50*c.* *Ans.*

2. A house, valued at $4000, was ensured against fire for 4½ per cent. a year; how much ensurance did the owner pay annually? *Ans.* $180.
3. What is the ensurance of a ship and cargo, valued at $87564, at 14¼ per cent.? *Ans.* $12477,87*c.*
4. What is the ensurance of a store of goods, valued at $7296, at 3½ per cent.? *Ans.* $255,36*c.*

COMMISSION OR FACTORAGE.

NOTE.—Commission or Factorage is an allowance of a certain per cent. from a merchant to his factor, or correspondent, engaged in buying or selling goods, &c.

EXAMPLES.

1. What is the commission on $1500, at 2½ per cent.? *Ans.* $37,50*c.*
2. What is the commission on $6000, at 1¾ per cent.? *Ans.* $105.
3. What is the commission on $3948, at 5 per cent.? *Ans.* $197,40*c.*
4. What is the commission on $1200, at 6½ per cent.? *Ans.* $78.

BROKERAGE.

NOTE.—Brokerage is an allowance of a certain per cent., to any person or persons who assist merchants or factors in buying and selling goods, &c

EXAMPLES.

1. What is the brokerage upon \$3000, at $1\frac{1}{2}$ per cent? *Ans.* \$45.

2. A broker sold goods amounting to \$4500, at $2\frac{1}{2}$ per cent; what was his demand against his employer? *Ans.* \$112,50.

3. If a broker sells goods to the amount of \$7500; what is his demand, at $3\frac{1}{4}$ per cent.? *Ans.* \$243,75*c*.

4. What is the brokerage upon \$5162, at $\frac{3}{4}$ per cent? *Ans.* \$38,71,5*m*.

BUYING AND SELLING STOCKS.

NOTE.—*Stock* is a name given to the capital, or funds of banks, trading companies, or of a fund established by governments, &c.; and the buying and selling of certain sums of money in those funds, the value of which often varies, are very usual.

EXAMPLES.

1. What is the value of \$400 of stock, at 106 per cent? *Ans.* \$424.

EXPLANATIONS.

The sum to be purchased must be multiplied by the rate per cent., as in Simple Interest, whether the stock be above or under par; that is, if \$100 of stock be worth *more* than \$100 of money, or *less* than \$100 of money; you must proceed precisely as in Simple Interest, multiply by the rate per cent., and divide by 100, or, rather, cut off two figures at the right hand, and the figures at the left will show the value of the stock in money.

$$\begin{array}{r} \$ \\ 400 \\ 106 \\ \hline \$424{,}00 \text{ Ans.} \end{array}$$

2. What is the value of \$3000 of bank stock, at 75 per cent? *Ans.* \$2250.

3. What is the value of \$6500 of capital stock in the United States Bank, at 112 per cent? *Ans.* \$7280.

4. What is the value of \$1000 of bank stock, at 95 per cent? *Ans.* \$950.

SINGLE RULE OF THREE DIRECT.

Q. What is the SINGLE RULE OF THREE DIRECT?

A. The Single Rule of Three Direct teaches, by having three numbers given to find a fourth, which shall have the same proportion to the third, as the second has to the first.

EXPLANATIONS.

The Single Rule of Three is merely an application of Multiplication and Division, as it is principally performed by those two rules in its operation. It is divided into two parts; the Rule of Three Direct, and the Rule of Three Inverse. On account of its extensive usefulness in the solution of nearly every mathematical inquiry, and also in the transaction of business, it is often called the Rule of Proportion, or, the Golden Rule of Proportion. The Rule of Three is so called, because three terms or numbers are given to find a fourth; and the principle upon which the rule is founded is this, that the result of any effect varies in proportion to the varying part of the cause; thus, the quantity or number of any article or articles, is in proportion to the money paid; and the space which is gone over by a uniform motion, is in proportion to the time. The sign of proportion is this, : :: : which should be placed between the numbers thus, 3 : 6 : : 8 : 16, and should be read thus, as 3 is to 6, so is 8 to 16. This is, you will readily perceive, very plain; for it is evident, that 16, the fourth term, bears the same proportion to 8, the third term, that 6, the second term, bears to 3, the first term.

Two of the given numbers, the first and second, are called terms of supposition, and the other term, the third given number, is called the term of demand. The terms of supposition are generally known by being preceded by *if*, *suppose*, &c. The term of demand is generally known by being preceded by the words, *how much*, *how many*, *how far*, *what will*, *what cost*, &c. When the third term is greater than the first, and requires the fourth term, or answer, to be greater than the second; or, when the third term is less than the first, and requires the fourth term, or answer, to be less than the second,

the sum belongs to, or must be worked by the Rule of Three Direct.

The Rule of Three *Direct* is much more useful for practical purposes, and in the transactions of business, than the Rule of Three *Inverse*, and is easily distinguished from it by the preceding conditions of the question; that is, whether more requires more, and less requires less; or, whether more requires less, and less requires more, &c.

All the following rules are worked by, and strictly belong to the Rule of Three Direct: Discount, Loss and Gain, Barter, Practice, Tare and Tret, Single and Double Fellowship, Alligation, Annuities, and also *Double* Rule of Three; for, as was stated on pages 139 and 140, these rules have different names, applicable to the different kinds of business to which they are applied, without involving any new *principle* in their operation, being all worked by the simple or fundamental rules only, as will be obvious to the learner on examination.

RULE.

Q. How do you state the terms in the Rule of Three Direct?

A. The term of supposition, which is of the same name or kind with the term of demand, must be written for the first term; and the remaining term of supposition must be written in the second place, which must always be of the same name or kind with the answer; and then the demanding term must be written in the third place, which must always be of the same name or kind with the first term.

Q. How do you work the different terms?

A. If the first and third terms are of different denominations, you must first reduce them to the lowest denomination mentioned in either; and if the second term stands in different denominations, you must also reduce it to the lowest denomination mentioned in that term. You must then multiply the second and third terms together, and divide their product by the first term, and the quotient will be the

answer, or fourth term, in the same denomination of the second term, in whatever denomination it stands, or has been reduced; and if the answer, or fourth term, does not stand in the highest denomination, you must bring it to the highest by Reduction Ascending.

Note.—To Teachers. The greatest difficulty and perplexity which learners experience in working this rule are, the stating of the questions or terms; and teachers should, therefore, require them to read the preceding and following explanations thoroughly and with attention, and exercise them frequently in stating questions. Teachers should continually explain the principles upon which the rules are founded, and trace them to the simple or fundamental rules on which they immediately depend. Teachers will be able, by pursuing this method, to free Arithmetick from all obscurity, and to convince the learner, that the whole of Arithmetick is merely the proper use and application of the simple or fundamental rules, as was stated on pages 139 and 140.

EXAMPLES.

1. If 6*lb.* of tea cost \$9, what will 12*lb.* cost? *Ans.* \$18.

EXPLANATIONS.

In this example, 12*lb.*, which moves the question, is the third term, 6*lb.*, the same kind is the first, and \$9 the second, the same kind as the answer. Here, more requires more, as it is very evident, that 12*lb.* will cost more than 6*lb.*, and, therefore, will require a greater sum than \$9 for the answer, or fourth term. Hence, this example belongs to this rule. You multiply the second and third terms together, and divide by the first, and the quotient is the answer. This example may be proved by stating it differently, or by inverting it: thus, if 12*lb.* of tea cost \$18, what will 6*lb.* cost? *Ans.* \$9.

```
lb.  $    lb.
 6 : 9 : : 12
     12
     ---
  6)108
     ---
    $18 Ans.
```

```
lb.  $    lb.
12 : 18 : : 6
      6
     ---
 12)108
     ---
     $9 Ans.
```

2. If 9*yd.* 1*qr.* of cloth cost £12 9*s.* 6*d.*, what will 18*yd* 2*qr.* cost? *Ans.* £24 19*s.*

EXPLANATIONS.

In this example, the first and third terms have the denominations of *yd.* and *qr.* You must, therefore, first reduce both terms to quarters, and then reduce the second term to pence, as pence is the lowest denomination mentioned in the second term. Then multiply the second and third terms together, that is, 2994*d.*, by 74*qr.*, and then divide the product by 37*qr.*, the first term, and the quotient will be pence, because the second denomination is reduced to pence. You must then reduce the pence to pounds by Reduction Ascending. When there is a remainder, and the second term has not been reduced to the lowest denomination, you must reduce it to the next lower denomination, and divide as before, and so on to the lowest denomination, or until nothing remains. You must be very particular in stating the sum, that the first and third terms be alike; that is, if the first term be yards, weights, &c., the third term must be yards, &c.; or, if the first term be money, the third term must be money. If the answer sought be yards, weights, or money, &c., then the second term must be the same, whether money, weights, or yards, &c.

```
yd.qr.   £  s. d.     yd.qr.
 9 1  :  12 9  6  ::  18 2
 4          20         4
 --         ---        --
37         249        74
            12
           ----
          2994
            74
          -----
         11976
        20958
        ------
   37)221556(5988d.
       185
       ---                 d.
        365           12)5988
        333              ----
        ---           2|0)49|9
         325             -----
         296          Ans. £24 19s.
         ---
          296
          296
```

By paying particular attention to the preceding EXPLANATIONS, you will be able to work any sum in the Rule of Three Direct.

3. If you can buy 7*lb.* of sugar for 75*c.*, how many pounds can you buy for $6? *Ans.* 56*lb.*

4. If 18*bu.* of wheat cost $24, what will 72*bu.* cost? *Ans.* $96.

5. If a firkin of butter, containing 56*lb.*, cost £1 17*s.* 4*d.*, what will 2*lb.* cost? *Ans.* 1*s.* 4*d.*

6. If 10 persons eat 3*bu.* of wheat in one month, how many bushels will 30 persons eat in the same time? *Ans.* 9*bu.*

7. If a man spend 75*c.* in 1*da.*, how much does he spend in a year? *Ans.* $273,75*c.*

8. If 3 sheep cost $3,33*c.*, what will 46 sheep cost? *Ans.* $51,06*c.*

9. If 9*yd.* of cloth cost $36, what will 12*yd.* cost? *Ans.* $48.

10. If 1*bu.* of wheat cost 75*c.*, what will 60*bu.* cost? *Ans.* $45.

11. If 22*yd.* of cloth cost $52,80*c.*, what will 5*yd.* 2*qr.* cost? *Ans.* $13,20*c.*

12. If 1*yd.* of muslin cost 29*c.*, what will 31*yd.* cost? *Ans.* $8,99*c.*

13. If a man pay $3,25*c.* a week for board, how many dollars is it a year? *Ans.* $169.

14. If a man receive 4*s.* 6*d.* a day, how much will he receive for 91 days? *Ans.* £20 9*s.* 6*d.*

15. If a man can earn $64,19*c.* in 15 weeks, how much can he earn in a year? *Ans.* $222,52,5*m.*

16. If 1*cwt.* of iron cost $6,86*c.*, what will 17*cwt.* 3*qr.* 12*lb.* cost? *Ans.* $122,50*c.*

17. If a man drinks 3*gi.* of spirits in 1 day, how much does he drink in a year? *Ans.* 34*gal.* 1*pt.* 3*gi.*

18. If a family of 7 persons consume 3*bu.* of potatoes in one month, how many bushels will serve a family of 21 persons? *Ans.* 9*bu.*

19. If 4½ tuns of hay will keep 3 cows over the winter, how many tuns will keep 25 cows the same time? *Ans.* 37½*T.*

20. What will 128*lb.* of cheese come to, at 8*c.* a pound? *Ans.* $10,24*c.*

21. If 1*lb.* of sugar cost 10*c.*, what will 6*cwt.* amount to? *Ans.* $67,20*c.*

22. If a piece of cloth, containing 39*yd.*, cost $350,22*c.*, what is it a yard? *Ans.* $8,98*c.*

23. What is the value of 6 gross of buttons, at 12*c.* 5*m.* a dozen? *Ans.* $9.

24. If you can buy 4*bu.* of corn for $3,36*c.*, how many bushels can you buy for $30,24*c.*? *Ans.* 36*bu.*

25. If 4*cwt.* of lead cost $13,44*c.*, what will 1*lb.* cost? *Ans.* 3*c.*

26. If 2*bu.* of salt cost $1,24*c.*, what will 425*bu.* cost? *Ans.* $263,50*c.*

27. If a staff 12*ft.* long cast a shade on level ground 9*ft.* long, what is the height of a steeple, whose shade is, at the same time, 186 feet? *Ans.* 248*ft.*

28. If 1*oz.* of tobacco cost 2*c.*, what will 17*cwt.* 3*qr.* 17*lb.* cost? *Ans.* $641,60*c.*

29. If the legislature of a state grant a tax of 8*m.* on the dollar, how much must that man pay who is $319,75*c*, on the list? *Ans.* $2,55,8*m.*

30. If 990 reams of paper cost $3564, what will 275 reams cost? *Ans.* $990.

31. If $100 gain $7 in a year, what will $650 gain in the same time? *Ans.* $45,50*c.*

32. The earth being 360 degrees in circumference, rolls round on its axis once in 24 hours, how long is it moving 1 degree? *Ans.* 4*min.*

33. At $5,50*c.* a thousand, what will 37 thousand of boards come to? *Ans.* $203,50*c.*

34. If 1*qr.* of beef cost $2,35*c.*, what will 16*cwt.* 3*qr.* cost? *Ans.* $157,45*c.*

35. If 1*lb.* of steel cost 12*c.*, what will 3*qr.* 26*lb.* cost? *Ans.* $13,20*c.*

36. A owes B $764, but B compounds with him for 62*c.* 5*m.*, what must he receive for his debt? *Ans.* $477,50*c.*

37. The President of the United States receives a salary of $25,000 a year, what is that a day? *Ans.* $68,49,3*m.*

38. If the yearly rent of a farm, containing 182*a.* be $273, what will be the rent of a part of it, containing 86*a.*? *Ans.* $129.

39. How many shingles will it require to cover the roof of a house, 44*ft.* in length, the rafters of which are 16*ft.*, allowing each shingle to cover 24 square inches? *Ans.* 8448 *shingles.*

40. If 17*T.* 2*hhd.* of wine cost $5468,40*c.*, what is it a pint? *Ans.* 15*c.* 5*m.*

41. If 1*bu.* of rye cost 59*c.*, what will 24 bushels cost? *Ans.* $14,16*c.*

42. A merchant bought 12 pieces of cloth, each containing 12 yards, at $2,80*c.* a yard, what did he pay for the whole? *Ans.* $403,20*c.*

43. How much is a farm worth, containing 225 acres, at $43,75*c*. an acre? *Ans.* $9843,75*c*.

44. If 24 cows require 96*bu*. of meal for three months, how many bushels will 48 cows require for the same time? *Ans.* 192*bu*.

45. If 147*lb*. of tea cost 165,37,5*m*., how much will 1*lb*. cost? *Ans.* $1,12,5*m*.

46. If a man spend $2,11*c*. a day, and his income be $890, 50*c*., how much will he save each year? *Ans.* 120,35*c*.

47. If 80 sheep cost $100, what will 90 sheep cost? *Ans.* $112,50*c*.

48. A bankrupt owes $1250, and his money and effects amount to $750,50*c*., how much will a creditor receive whose demand is $80. *Ans.* $48,03,2*m*.

49. If 65*gal*. of water fall into a cistern, containing 378*gal*., in one hour, and by a leak in the cistern 15*gal*. leak out in 20 minutes, in what time will the cistern be filled? *Ans.* 18*h*. 54*min*.

50. The valuation of the property in a certain town, according to the town's inventory, is $305,000, and the tax levied on that town is $1525, what is that person's tax, whose estate, according to the inventory, is valued at $750? *Ans.* $3,75*c*.

51. A merchant bought 4*hhd*. of brandy for $475; he paid for the freight $50, for the duties and other charges $25, and he wishes to gain $50, how must he sell it per hogshead? *Ans.* $150.

52. A and B depart from the same place, and travel the same road, but A goes 7 days before B, at the rate of 24 miles a day; B follows at the rate of 30 miles a day, how far will they travel before B overtakes A? *Ans* 840*m*.

53. If 150*lb*. of coffee cost $18, what will 18*lb*. cost? *Ans.* $2,16*c*.

54. A merchant bought 16 hogs, weighing together 3752*lb*., at $5,50*c*. a hundred-weight, what did he pay for the whole? *Ans.* $206,36*c*.

55. The time made out by the inhabitants of a certain school district, during the winter, amounted to 6080 days, and the teacher received $150 for his services, what did that man pay who had sent 160 days to the school? *Ans.* $3,94,7*m*.

RULE OF THREE INVERSE.

Q. What is the Rule of Three Inverse?

A. The Rule of Three Inverse teaches, by having three numbers given to find a fourth, which shall have the same proportion to the second, as the first has to the third.

EXPLANATIONS.

The Rule of Three Inverse, like the Rule of Three Direct, is merely an application of Multiplication and Division. The only difference, or distinction between Rule of Three Direct and Inverse, is, that in direct proportion more requires more, or less requires less in every question belonging to that rule; but, in inverse proportion, more requires less, and less requires more. More requiring less, is when the third term is greater than the first, and requires the fourth term, or answer, to be less than the second. Less requiring more, is when the third term is less than the first, and requires the fourth term, or answer, to be greater than the second. The greatest difficulty, and, in fact, the only difficulty, which you will have to encounter, will be to distinguish *inverse* from *direct* proportion; but if you will pay particular attention to the rule for stating the questions, which is the same in both, you will, with the greatest ease, be able to decide to which it belongs, after having considered whether *less* requires *more*, or *more* requires *less.*

RULE.

Q. How do you state and work the questions in Rule of Three Inverse?

A. The questions must be stated, and the terms reduced, if of different denominations, the same as in Rule of Three Direct. You must then multiply the first and second terms together, and divide the product by the third, and the quotient will be the answer, in a denomination of the same name to which the second term was reduced.

EXAMPLES.

1. If 6 men can build a house in 96 days, in how many days can 72 men build it? *Ans.* 8*da.*

EXPLANATIONS.

men.	*da.*	*men.*
6 :	96 : :	72
	6	
72)	576(8*da.A.*	
	576	

In this example, 72 men, which moves the question, is the third term, 6 men, the same kind, is the first, and 96 days, the second, the same as the answer. Here, more requires less, and it is, therefore, very evident, from the conditions of the question, that this sum belongs to the Rule of Three *Inverse;* for, it is perfectly plain, that the more men there are employed, the less time it will require to build the house. In this example, also, the third term is larger than the first, and requires the fourth term, or answer, to be less than the second; and it is very evident, that the fourth term, or answer, should be only one twelfth part as large as the second term, because it must require only one twelfth the number of days for 72 men to do a piece of work, that it would require 6 men to do the same piece of work. It will also appear, that the fourth term, or answer, 8 days, bears the same proportion to the second term, 96 days, that the first term, 6 men, bears to the third term, 72. You must, therefore, multiply the first and second terms together, and divide by the third, and the quotient will be the answer.

2. If $100, in 12*mo.* bring $6 interest, what sum, or principal, will bring the same in 8*mo.*? *Ans.* $150.

3. If 30*bu.* of grain, at 50*c.* a bushel, will pay a debt, how many bushels, at 75*c.* a bushel, will pay the debt? *Ans.* 20*bu.*

4. If 8 men can mow a piece of meadow in 24 days, how many men can do it in 4 days? *Ans.* 48 *men.*

5. If I lend my friend $100 for 180 days, how long ought he to lend me $450 to return the kindness? *Ans.* 40*da.*

6. If a traveller performs a journey in 10 days, when the day is 12 hours long, in how many days would he perform the same journey, when the day is but 8 hours long? *Ans.* 15*da.*

7. How much land, at $2,50*c.* an acre, should be given in exchange for 360 acres, at $3,75*c.* an acre? *Ans.* 540*a.*

8. A garrison of 1200 men has provisions for 9 months, at

the rate of 14*oz.* per man a day, how long will the provisions last, at the same allowance, if the garrison be re-enforced by 400 men? *Ans.* 6¾*mo.*

9. The imperial canal, which intersects China from north to south, employed 30,000 men 43 years in its construction, how many men must have been employed to have completed it in 1 year? *Ans.* 1,290,000 *men.*

10. If, when wheat is 83*c.* a bushel, the cent loaf weighs 9*oz.*, how many ounces should it weigh when wheat is $1,24,5*m.* a bushel? *Ans.* 6*oz.*

11. How many yards of paper, 3*qr.* wide, will paper a room that is 24*yd.* round, and 4*yd.* high? *Ans.* 128*yd.*

12. If, for $6, a merchant has 4*cwt.* of goods carried 160*m.*, how far can he have 20*cwt.*, or 1 tun, carried for the same money? *Ans.* 32*m.*

13. How many yards of baize, 3*qr.* wide, will line a cloak, which has in it 12*yd.* of camlet, ½ a yard wide? *Ans.* 8*yd.*

14. If a board be 9*in.* in width, how much in length will make a square foot? *Ans.* 16*in.*

15. How many yards of sarcenet, that is 3*qr.* wide, will line 18*yd.* of cloth, 2*yd.* wide? *Ans.* 48*yd.*

16. A cistern has a pipe which will empty it in 15 hours, how many pipes of the same capacity will empty it in 45*min.*? *Ans.* 20 *pipes.*

DISCOUNT.

Q. What is Discount?

A. Discount is an allowance made for the payment of any sum of money before it becomes due, or upon advancing ready money on notes, bills, obligations, &c., which are payable at some future day or period.

EXPLANATIONS.

As was stated on page 168, Discount is only an application of the Rule of Three Direct, and has this name given to it merely as applicable to its application and use in this particular transaction of business.

RULE.

Q. How do you state and work the terms to find the discount of any given sum?

A. As the amount of $100, or £100, at the given rate and time, is to the interest of $100, or £100, at the same rate and time, so is the given sum to the discount. To find the present worth of a given sum, you must substract the discount from the given sum, and the remainder will be the present worth, or such a sum as if put to interest would, in the given time, and at the given rate per cent., amount to the sum or debt then due. Or, to find the present worth of any given sum, you may state it thus: as the amount of $100, or £100, is to $100, or £100, so is the given sum or debt to the present worth.

EXAMPLES.

1. What is the discount of $100, due 1 year hence, at 7 per cent. a year? *Ans.* $6,54,2*m.*

EXPLANATIONS.

In this example, you multiply the given sum, $100, by 7, the rate per cent., and divide the product by $107, the amount of $100 and the rate per cent., $7, added together, and the quotient is the answer, or discount of $100 for 1 year at 7 per cent.

```
  $     $       $
 107 :  7  : : 100
           100
           ———      $ c. m.
      107)700(6,54,2 discount.
          642
          ———
           580
           535
           ———
            450
            428
            ———
             220
             214
             ———
               6
```

2. What is the present worth of $100, due 1 year hence, at 7 per cent.? *Ans.* $93,45,7*m*.

EXPLANATIONS.

In this example, as before, multiply the second and third terms together, and divide the product by the first term, and the quotient will be the present worth of $100, due a year hence, at 7 per cent. You will at once perceive, that $100, the second term, is the present worth of $107, due a year hence; for $100 put to interest at 7 per cent., in one year, amounts to $107. It is also perfectly evident, that the fourth term, or answer, bears the same proportion to $100, the third term, that $100, the second term, bears to $107, the first term.

```
  $     $      $
107 : 100 : : 100
      100
      ———    $ c.m.
107)10000(93,45,7 present worth.
     963
     ———
      370
      321
      ———
       490
       428
       ———
        620
        535
        ———
         750
         749
         ———
         101
```

By the preceding examples and EXPLANATIONS, you will readily see, that, in discount, money is supposed not to bear interest until it is due; and it is perfectly reasonable and just, that a discount should be made for the payment of money before it is due; for the debtor, by retaining the money until it is due, may put it to interest for the time; but, should he pay it before it becomes due, he will give that advantage and benefit to another. Many persons have very incorrectly supposed, that the *interest* on any given sum, for a given rate and time, was the *discount*, and that this interest, taken from the given sum, or principal, would give the present worth. This supposition is, however, very erroneous; for, if that were true, the discount on $100 would be $7, and the present worth of $100, due, or payable, one year hence, at 7 per cent., would be $93; but $93 put to interest for one year, will not amount

to $100, which makes it perfectly evident, that it is entirely wrong to suppose, that the *interest* and *discount* of any given sum, for any given rate and time, are the same.

3. What is the discount of $300, due 2 years hence, at 6 per cent.? *Ans.* $32,14,2*m*.
4. What is the discount of $595, due 2 years hence, at 7 per cent.? *Ans.* $73,07*c*.
5. What is the discount on $2000, due 4 years hence, at 7 per cent.? *Ans.* $437,50*c*.
6. What is the present worth of $600, due 4 years hence, at 5 per cent.? *Ans.* $500.
7. What is the present worth of $500, due 2*yr*. and 8*mo*. hence, at 7 per cent.? *Ans.* $421,35*c*.
8. What is the present worth of a note, for $345,50*c*., due 10*mo*. hence, at 7 per cent.? *Ans.* $326,45,7*m*.
9. A merchant bought goods, amounting to $615,75*c*., at 6 months' credit, how much ready money must he pay, if a discount of 4½ per cent. be allowed? *Ans.* $602,20*c*.
10. What is the difference between the interest of $1204, at 5 per cent., for 8 years, and the discount of the same sum, for the same time and rate per cent.? *Ans.* $137,60*c*.

NOTE.—When sundry sums are to be paid at different times, you must find the present worth of each payment separately, and then add them into one sum.

11. What is the present worth of $2000, of which $500 are due in 6*mo*., $800 in 1*yr*., and the balance, $700, in 3*yr*., at 6 per cent.? *Ans.* $1833,37,4*m*.
12. What is the present worth of $1600, one half due in 1*yr*., and the other half in 2*yr*., at 7 per cent.? *Ans.* $1449,41,6*m*.
13. What is the present worth of $2000, one half due in 9 months, and the other half in 2 years, at 6 per cent.? *Ans.* $1849,79,6*m*.

NOTE.—But when discount for the present payment of notes, obligations, &c., for money is made, without regard to time, it is then found precisely as the interest of the given sum for one year.

14. What is the discount of $476, at 6 per cent.? *Ans.* $28,56*c*.

15. What is the discount of $853, at 4 per cent.? *Ans.* $34,12*c.*

16. A merchant bought goods on credit, amounting to $1656, but by paying ready money he has a discount of 5 per cent. allowed, how much ready money must he pay? *Ans.* $1573,20*c.*

LOSS AND GAIN.

Q. What is Loss and Gain?

A. Loss and Gain teaches merchants and traders the knowledge of what is gained or lost in buying or selling goods, produce, &c.; and it also teaches them to raise or fall on the price of their goods, &c., so as to gain or lose so much per cent., &c.

EXPLANATIONS.

Loss and Gain is merely a particular application of the Rule of Three Direct, as was stated on page 168; and I wish you continually to bear in mind, that in the operation of all these rules, there is no new *principle* involved, but that the particular *application* of the fundamental rules, gives each rule its peculiar name.

RULE.

Q. How do you state and work the terms to find what is gained or lost per cent.?

A. You must first find what the gain or loss is by Substraction; that is, if the gain be required, substract the price it cost from the price for which it was sold, and the remainder, or difference, will show the gain on the sum first expended; and, if the loss be required, substract the price for which it was sold from the price which was paid for it, and the remainder

or difference will show the loss on the sum first expended. Then, as the price it cost is to the gain or loss, so is \$100, or £100, to the gain or loss per cent.

EXAMPLES.

1. A merchant bought coffee at 22*c.* a pound, and sold it for 33*c.* a pound, what did he gain per cent., or what did he gain in buying to the amount of \$100, at that rate? *Ans.* \$50, or 50 per cent.

EXPLANATIONS.

In this example, you first substract 22*c.*, the price that one pound of coffee cost, from 33*c.*, the price for which it was sold; and then say, as 22*c.* is to 11*c.*, so is \$100 to the given per cent.

```
 c.            c.   c.       $  c.
33 sold.      ,22 : ,11 : :  100,00
22 bought.                    11
—                     c.   ———————   $  c.
11 gain a lb.        ,22)1100,00(50,00 Ans.
                         110
                         ———
                          000
```

You will readily perceive, that the merchant, on the sale of one pound, gained half what it cost him, because 11*c.*, the gain, is half of 22*c.*, the cost; and, therefore, he must have gained \$50 in buying to the amount of \$100, as \$50 is half of \$100; and the gain on \$100 must be in the same proportion as the gain on 22*c.*

2. A merchant bought a piece of velvet, at \$2 a yard, and sold it for \$1,50*c.*, what did he lose per cent.? *Ans.* 25 per cent.

EXPLANATIONS.

In this example, you first substract \$1,50*c.*, the price for which one yard of velvet was sold, from \$2, the price it cost; and then say, as \$2 is to 50*c.*, so is \$100 to the loss per cent.

```
$ c.               $ c.   c.       $  c.
2,00 bought.      2,00 : ,50 : :  100,00
1,50 sold.                          50
—                                 ———————
,50 loss a yd.          2 | 00)5000 | 00
                                  ———————
                                 $25 Ans.
```

It is perfectly evident, that he lost 25 per cent., because 50*c.* is one fourth of \$2, and \$25 is one fourth of \$100.

3. If a grocer buys butter, at 12c. a pound, and sells it at 16c., how much does he gain per cent.? *Ans.* $33\frac{1}{3}$ per cent.

4. A merchant bought 4 pieces of cloth, each piece containing 30 yards, at $1,25c. a yard, but on examining it he found one piece so badly damaged that he could not sell it, how much per yard must he sell the remainder to gain $10 on the whole? *Ans.* $1,77,7m.

RULE.

Q. How do you state and work the terms to find how a commodity must be sold, to gain or lose so much per cent.?

A. As $100, or £100, is to the price it cost, so is $100, or £100, with the profit added, or the loss substracted, to the gaining or selling price.

EXAMPLES.

1. If a merchant buys calico at 24c. a yard, how must he sell it a yard to gain 25 per cent.? *Ans.* 30c.

EXPLANATIONS.

To gain 25 per cent., is, as you will readily perceive, adding one fourth the given sum or cost to itself; and, therefore, as the third term is increased by the per cent. above the first term, so the fourth term, or answer, must increase above the second.

```
  $      c.     $
100 : ,24 : : 125
               24
              ---
              500
             250
          -------
  1 | 00)30 | 00
        --
        30
```

2. If a merchant buys wheat at 50c. a bushel, how must he sell it to gain 10 per cent.? *Ans.* 55c.

3. A merchant bought a piece of cloth, at $2 a yard, which proved to be not as good as he expected, and he is willing to lose $12\frac{1}{2}$ per cent., how must he sell it a yard? *Ans.* $1,75c.

4. A merchant bought a hogshead of molasses, at 50c. a gallon, which he found to be of an inferiour quality, so that he must lose 10 per cent., what will it then be a gallon? *Ans.* 45c.

S H F

RULE.

Q. How do you state and work the terms to find what the commodity cost, when there is gain or loss per cent. ?

A. As $100, or £100, with the gain per cent. added, or the loss per cent. substracted, is to the price, so is $100, or £100 to the first cost.

EXAMPLES.

1. If a merchant, by selling tea at 87*c.* 5*m.* a pound, loses $12,50*c.* per cent., what did it cost a pound? *Ans.* $1.

EXPLANATIONS.

In this example, you first substract $12,50*c.*, the loss per cent., from $100; and then say, as $87,50*c.* is to 87*c.* 5*m.*, the price for which it is sold, so is $100 to the price it cost.

```
 $  c.     c. m.      $  c.
87,50 : ,87,5 : : 100,00
                  10000
                 ------
87,5 | 0)875000 | 0(1,00,0
         875
         ---
         000
```

2. If a merchant, by selling cloth at $3 a yard, gains 20 per cent., what did the cloth cost a yard? *Ans.* $2,50*c.*

3. If a merchant, by selling broadcloth at $3,25*c.* a yard, loses 20 per cent., what did the cloth cost a yard? *Ans.* $4,06,2¼*m.*

4. A merchant sold 80*lb.* of chocolate at 25*c.* a pound, and gained 9 per cent., what did the whole cost him? *Ans.* $18,34,8*m.*

RULE.

Q. How do you state and work the terms, when, if commodities be sold at a given rate there is so much gained or lost per cent., to find what would be gained or lost per cent., if sold at another rate?

A. As the first price is to $100, or £100, with the profit per cent. added, or the loss per cent. substracted, so is the other price, to the gain or loss per cent., at the other rate or price.

NOTE.—If your answer exceed $100, or £100, the excess is the gain per cent., but if it be less than $100 or £100, that deficiency is the loss per cent.

EXAMPLES.

1. A merchant sold *4cwt.* of sugar for $32, and lost 12 per cent., how much per cent. would he have gained or lost, if he had sold the same sugar for $36? *Ans.* 1 per cent. loss.

2. A grocer sold brandy at $1,12*c.* a gallon, and gained 20 per cent., how much per cent. would he have gained if he had sold it at 95*c.* a gallon? *Ans.* $1,78,5*m.* per cent.

3. A merchant sold potash at $125 a tun, and gained 10 per cent., how much per cent. would he have gained if he had sold it at $150 a tun? *Ans.* 32 per cent.

BARTER.

Q. What is BARTER?

A. Barter teaches merchants and traders to exchange one commodity, or specifick article for another, agreeably to the price or value agreed upon by the parties concerned, so that neither party shall sustain loss.

EXPLANATIONS.

Barter, as was stated on page 168, is merely an application of the Rule of Three Direct, and has this particular name as applicable to the *exchange* of specifick articles.

RULE.

Q. How do you state and work the terms in the Rule of Barter?

A. First find the value of the commodity, whose quantity is given; then find what quantity of the other, at the proposed rate, can be purchased for the same money, and it will give the answer

EXAMPLES.

1. How much rye, at 70*c.* a bushel, must be given for 28 bushels of wheat, at \$1,25*c.* a bushel? *Ans.* 50 bushels of rye.

EXPLANATIONS.

```
$ c.                             c. bu.   $ c.
1,25 wheat a bu.                 ,70 : 1 :: 35,00
  28 num. of bu.                               1
----                                       -----
1000                                   70)3500(50bu. Ans.
250                                       350
----                                       ---
$35,00 price of 28bu. wheat.                 0
```

You must first find the amount or price of 28*bu.* of wheat, at \$1, 25*c.*, which is \$35. Then say, as 70*c.*, the price of 1*bu.* of rye, is to 1*bu.*, so is \$35, the price of 28*bu.* of wheat, to the answer, or fourth term, which is 50*bu.* of rye. The principle of this operation is very evident, for it is perfectly plain, that for as many times as there are 70*c.* contained in \$35, so many bushels of rye can be had in exchange for 28 bushels of wheat.

2. How much sugar, at 8*c.* a pound, must be given for 20*cwt.* of tobacco, at \$7,50*c.* a *cwt.*? *Ans.* 16*cwt.* 2*qr.* 27*lb.*

3. How much tea, at 64*c.* a pound, must be given for 448*lb.* of coffee, at 20*c.* a pound? *Ans.* 140*lb.*

4. How much wool, at 30*c.* a pound, must be given for 125*lb.* of flax, at 12*c.* a pound? *Ans.* 50*lb.*

5. A merchant had 1286*yd.* of linen, at 43*c.* a yard, for which he received 2*cwt.* 1*qr.* 13*lb.* of chocolate, at 14*c.* a pound, and the balance in money, how much money did he receive? *Ans.* \$515,88*c.*

6. A had 41*cwt.* of hops, at \$4,50*c.* a hundred-weight, for which B gave him \$28,50*c.* in money, and the balance in salt, at 80*c.* a bushel, how many bushels of salt did he receive? *Ans.* 195*bu.*

7. How many hogsheads of brandy at 6*s.* 8*d.* a gallon must be given for 126*yd.* of cloth, at 10*s.* a yard? *Ans.* 3*hhd.*

8. D has calico worth 20*c.* a yard, ready money, but in barter he will have 25*c.*; E has broadcloth worth \$2,50 a yard, ready money; at what price should the broadcloth be rated in barter? *Ans.* \$3,12,5*m.*

9. G had 5 pieces of calico, each piece containing 95*yd.*, at 23*c.* a yard, for which H gave him 32*yd.* of broadcloth, at $2,50*c.* a yard, and the balance in rye flour, at $1,50*c.* a hundred-weight, how many hundred-weight of flour did G receive? *Ans.* 19*cwt.* 2*qr.*

PRACTICE.

Q. What is PRACTICE?

A. PRACTICE is a contraction of the Rule of Three Direct, when the first term is a unit or one, and is a concise method of resolving most questions that occur in trade or business, where money is reckoned in pounds, shillings, and pence.

EXPLANATIONS.

Practice has its name from its frequent and daily use among merchants and traders. This Rule, which was formerly of great use among merchants and tradesmen, when the price of one was given in pounds, shillings, and pence, or sterling money, is now rendered quite useless, as reckoning in federal money has become very general; and it is much more easy to work by multiplication, when the price of a unit or one is given in federal money. I have, therefore, given but few examples in this rule.

RULE.

Q. How do you find the amount or price of a given quantity of yards, pounds, &c., when the price of one yard or pound is given in shillings?

A. The given number of yards or pounds must be supposed to be so many pounds in money, and aliquot parts of a pound must be taken: thus, if the price be 5s., you must take one fourth; if 10s., take one half; if 15s., take three fourths, or take one

half and one fourth and add the quotients: if there be $\frac{1}{2}$ of a yard or pound, you must call it 10*s*., if $\frac{1}{4}$ of a yard, call it 5*s*.; if $\frac{3}{4}$, call it 15*s*., in stating or setting down the given sum for operation.

EXAMPLES.

1. What is the value of 980*yd.* of cloth, at 10*s.* a yard? *Ans.* £490.

EXPLANATIONS.

In this example, you set down the 980*yd.* as so many pounds in money; and then divide it by 2 or $\frac{1}{2}$; for it is perfectly evident, that at 10*s.* a yard, one half of the number of yards would be the value of the cloth in pounds. I have also worked this example by the Rule of Three Direct, to show you that Practice, as I have before told you, is a contraction of that rule.

$$\begin{array}{rl} s. & \pounds \\ 10 = \frac{1}{2}) & 980 \\ \hline & \pounds 490\ \textit{Ans.} \end{array}$$

Rule of Three Direct.

$$\begin{array}{r} yd.\quad s.\quad\ \ yd. \\ 1 : 10 :: 980 \\ 10 \\ \hline 2\,|\,0)980\,|\,0 \\ \hline \pounds 490\ \textit{Ans.} \end{array}$$

2. What is the value of 490*lb.* of tea, at 15*s.* a pound? *Ans.* £367 10*s.*
3. What is the value of 792*yd.* of cloth, at 5*s.* a yard? *Ans.* £198.
4. What is the value of 123*bu.* of wheat, at 10*s.* a bushel? *Ans.* £61 10*s.*
5. What is the value of 687½*yd.* of cloth, at 5*s.* a yard? *Ans.* £171 17*s.* 6*d.*

RULE.

Q. How do you find the amount or price of a given quantity of yards, pounds, &c., when the price of one yard or pound is given in pence?

A. The given number of yards or pounds must be supposed to be so many shillings, and aliquot parts of a shilling must be taken; thus, if the price be 3*d.*

a yard or pound, you must take one fourth; if 6*d.*, take one half; if 9*d.*, take three fourths, or take one half and one fourth and add the quotients: if there be $\frac{1}{2}$ of a yard or pound, you must call it 6*d.*; if $\frac{1}{4}$ of a yard, call it 3*d.*; if $\frac{3}{4}$, call it 9*d.*, in stating or setting down the given sum for operation.

EXAMPLES.

1. What is the value of 1276*yd.* of riband, at 3*d.* a yard? *Ans.* £15 19*s.*

EXPLANATIONS.

In this example, you divide by 4 or $\frac{1}{4}$, as it is evident, that at 3*d.* a yard, one fourth of the number of yards would be the value of the riband in shillings; and you then divide the amount by 20 to bring it to pounds.

$$\begin{array}{rl} d. & s. \\ 3=\frac{1}{4}) & 1276 \\ \hline 2|0) & 31 \mid 9 \\ \hline & £15\ 19s. \end{array}$$

2. What is the value of 792$\frac{1}{4}$*lb.* of sugar, at 9*d.* a pound? *Ans.* £29 14*s.* 4*d.* 2*qr.*
3. What is the value 112*lb.* of rice, at 6*d.* a pound? *Ans.* £2 16*s.*
4. What is the value of 1728*yd.* of tape, at 4*d.* a yard? *Ans.* £28 16*s.*
5. What is the value of 912*yd.* of riband, at 5*d.* a yard? *Ans.* £19.

RULE.

Q. How do you find the amount or price of a given quantity of yards, pounds, &c., when the price of one yard or pound is given in farthings?

A. The given number of yards or pounds must be supposed to be so many pence, and aliquot parts of a penny must be taken: thus, if the price be one farthing a yard or pound, you must take one fourth; if two farthings, take one half; if three farthings, take three fourths, or take one half and one fourth

and add the quotients: if there be ½ of a yard or pound, you must call it 2 farthings; if ¼ of a yard, you must call it one farthing; if ¾, call it 3 farthings, in stating or setting down the given sum for operation.

EXAMPLES.

1. What is the value of 484 skeins of silk, at 2 farthings a skein? *Ans.* £1 0*s.* 2*d.*

EXPLANATIONS.

In this example, you set down the 484 skeins as so many pence, and first divide it by 2 or ½; for it is perfectly evident, that at 2 farthings a skein, one half of the number of skeins would be the value of the silk in pence; and you then divide by 12 to bring the amount into shillings, and by 20 to bring it to pounds.

```
qr.      d.
2 = ½)484
     ------
  12)242—2
     ------
 2|0)2|0
     ------
Ans. £1 0s. 2d.
```

2. What is the value of 240*yd.* of tape, at 1 farthing a yard? *Ans.* 5*s.*

3. What is the value of 793*yd.* of riband, at 3 farthings a yard? *Ans.* £2 9*s.* 6*d.* 3*qr.*

4. What is the value of 17482½*yd.* of tape, at 2 farthings a yard? *Ans.* £36 8*s.* 5*d.* 1*qr.*

5. What is the value of 69384*yd.* of cord, at 1 farthing a yard? *Ans.* £72 5*s.* 6*d.*

TARE AND TRET.

Q. What is Tare and Tret?

A. Tare and Tret are allowances made by the seller to the buyer, on the weight of some particular articles, goods, or commodities.

Tare is an allowance made for the weight of the

barrel, bag, or box, or whatever contains the articles, goods, or commodities.

TRET is an allowance of 4*lb.* in every 104*lb.* for waste, dust, &c. *Gross* weight is the weight of the goods, together with the barrel, bag, or box, or whatever contains the articles or commodities.

In addition to the preceding allowances, there is an allowance on some commodities, of 2*lb.* in every 3*cwt.*, to make the weight hold out, when goods are re-weighed, claimed only by particular merchants.

Suttle is what remains after a part of the allowance is deducted from the *gross* weight. *Nett* weight is what remains after all allowances are deducted.

EXPLANATIONS.

As was stated on page 168, the questions or sums in this rule may be worked by the Rule of Three Direct, and also may be, like the Rule of Three Direct, reduced to Multiplication and Division. This rule is, however, like Practice, more properly a contraction of the Rule of Three Direct, and it is, therefore, generally more convenient and proper to work the questions by taking aliquot parts.

RULE.

Q. How do you find the nett weight when the tare is given on the whole gross weight?

A. The tare must be substracted from the gross, and the remainder will be the nett weight.

EXAMPLES.

1. What is the nett weight of a hogshead of sugar, weighing 8*cwt.* 3*qr.* 17*lb.* gross, tare 3*qr.* 16*lb.*? *Ans.* 8*cwt.* 0*qr.* 1*lb.*

EXPLANATIONS.

In this example, you merely deduct the tare, 3*qr.* 16*lb.*, from the gross, 8*cwt.* 3*qr.* 17*lb.*, and the remainder or difference is the nett weight. This operation is performed, as you will readily perceive, simply by Compound Substraction.

cwt.	*qr.*	*lb.*	
8	3	17	gross.
	3	16	tare.
8	0	1	nett weight.

2. What is the nett weight of 99*cwt.* 3*qr.* 18*lb.* gross, tare 2*cwt.* 3*qr.* 20*lb.*? *Ans.* 96*cwt.* 3*qr.* 26*lb.*

3. What is the nett weight of 4 hogsheads of sugar the gross weight, and tare, as follows?

	cwt.	*qr.*	*lb.*
No. 1	3	3	2
" 2	4	0	19
" 3	4	1	10
" 4	4	0	0
Gross weight,	16	1	3
Whole tare,	1	0	20
Ans.	15	0	11 nett weight.

	qr.	*lb.*
Tare	1	1
"	1	4
"	1	8
"	1	7

Whole tare 1*cwt.* 0*qr.* 20*lb.*

RULE.

Q. How do you find the nett weight when the tare is so much a hundred-weight?

A. First find the tare, by taking aliquot parts of a hundred-weight, as in Practice; then deduct the tare from the gross, and the remainder, or difference will be the nett weight: or, work the questions by the Rule of Three Direct; thus, as 112*lb.* is to the tare a *cwt.*, so is the given weight to the tare; then deduct the tare from the gross weight, and the remainder will be the answer.

EXAMPLES.

1. What is the nett weight of 6*hhd.* of sugar, gross weight 51*cwt.* 3*qr.*, tare 16*lb.* a hundred-weight? *Ans.* 44*cwt.* 1*qr.* 12*lb.*

EXPLANATIONS.

In this example, you divide the gross weight by 7, because 16*lb.* is $\frac{1}{7}$ of a hundred-weight, and then deduct that quotient from the gross weight, and the remainder or difference will be the nett weight. Or you may work the sum, as

lb.	*cwt.*	*qr.*	*lb.*	
16 $=\frac{1}{7}$)	51	3	0	gross weight.
	7	1	16	tare deducted.
Ans.	44	1	12	nett weight.

Rule of Three Direct.

112 : 16 : : 51 3

here stated, by the Rule of Three Direct; as 112lb. is to 16lb., so is 51cwt. 3qr. to the nett weight; and deduct the quotient or fourth term from the gross weight or third term, and the remainder will be the nett weight. But this operation is much more tedious and inconvenient than to take aliquot parts, and should not, therefore, be practised.

2. What is the nett weight of 84cwt. 2qr. 14lb. gross, tare 14lb. a hundred-weight? *Ans.* 74cwt. 0qr. 5lb. 4oz.

3. What is the nett weight of 9hhd. of tobacco, each weighing 8cwt. 3qr. 14lb. gross, tare 16lb. a hundred-weight? *Ans.* 68cwt. 1qr. 24lb.

4. What is the weight of 57cwt. 2qr. 24lb. gross, tare 7lb. a hundred-weight? *Ans.* 54cwt. 0qr. 12lb.

RULE.

Q. How do you find the nett weight when tret is allowed with the tare?

A. First find the tare as in the preceding rule, deduct that from the gross, and call the remainder suttle: then divide the suttle by 26, as 26 is the $\frac{1}{4}$ part of 104, and the quotient will be the tret, which deduct from the suttle, and the remainder will be the nett weight.

EXAMPLES.

1. What is the nett weight of 3 tierces of rice, weighing 14cwt. 2qr. 14lb. gross, tare 16lb. a hundred-weight, tret 4lb. per 104lb.? *Ans.* 12cwt. 0qr. 6lb.

EXPLANATIONS.

In this example, you first divide the gross by 7, as 16lb. is $\frac{1}{7}$ part of 112lb., and deduct the quotient from the gross, and the remainder is the suttle: then divide the suttle by 26, as 4lb. is $\frac{1}{26}$ part of 104lb.; and then deduct the quotient from the suttle, and the remainder will be the nett weight.

lb.		cwt.	qr.	lb.	
$16=\frac{1}{7}$		14	2	14	gross.
		2	0	10	tare.
$4=\frac{1}{26}$	)	12	2	4	suttle.
		0	1	26	tret.
	Ans.	12	0	6	nett weight.

2. What is the nett weight of 17*hhd.* of sugar, weighing 120*cwt.* 2*qr.* gross, tare 1*cwt.* 2*qr.* 8*lb.*, tret 4*lb.* per 104*lb.*? *Ans.* 114*cwt.* 1*qr.* 12*lb.*

3. What is the nett weight of 247*cwt.* 2*qr.* 15*lb.* gross, tare 28*lb.*, a hundred-weight, tret 4*lb.* per 104*lb.*? *Ans.* 178*cwt.* 2*qr.* 9*lb.* 4*oz.*

RULE.

Q. How do you find the nett weight when tare, tret, and cloff are allowed?

A. First find and deduct the tare and tret as in the preceding rule, and divide the suttle by 168, as 2*lb.* is the $\frac{1}{168}$ part of 3*cwt.*, the quotient will be the cloff, which deduct from the suttle, and the remainder will be the nett weight.

EXAMPLES.

1. What is the nett weight of 3*hhd.* of tobacco, weighing 4689*lb.* gross, tare 321*lb.*, tret 4*lb.* per 104*lb.*, and cloff 2*lb.* per 3*cwt.*? *Ans.* 4175*lb.*

EXPLANATIONS.

First deduct the tare from the gross; then divide the suttle by 26, to obtain the tret, as before directed, and deduct the quotient from the suttle, and then divide that remainder by 168, as 2*lb.* is $\frac{1}{168}$ part of 3*cwt.*, and deduct the quotient from the suttle, and the remainder will be the nett weight.

	lb.	
	4689	whole gross.
	321	tare.
lb.	——	
4 $=\frac{1}{26}$)	4368	suttle.
	168	tret.
lb.	——	
2 $\frac{1}{168}$ of 3*cwt.*)	4200	suttle.
	25	cloff.
	——	
Ans.	4175	nett weight.

2. What is the nett weight of 4*hhd.* of sugar, each weighing 8*cwt.* 1*qr.* 10*lb.* gross, tare 14*lb.* a hundred-weight, tret 4*lb.* per 104*lb.*, and cloff 2*lb.* per 3*cwt.*? *Ans.* 27*cwt.* 3*qr.* 16*lb.* 10*oz.*

SINGLE FELLOWSHIP.

Q. What is SINGLE FELLOWSHIP?

A. Single Fellowship is a rule by which merchants, &c., trading in company with a joint stock, are enabled to ascertain each person's particular share of the gain or loss, in proportion to his share in the joint stock, each share of which stock having been in trade an equal term of time.

EXPLANATIONS.

As was stated on page 168, Single and Double Fellowship is only an application of the Rule of Three Direct, and has this name given to it merely as applicable to its application and use in this particular transaction of business. By this rule also, legacies are adjusted, and the effects of bankrupts divided among their creditors; for, as the sum due to the creditors is to the bankrupt's estate, so is each creditor's demand to his share of the bankrupt's estate.

RULE.

Q. How do you state and work the terms to find each man's share of the gain or loss?

A. As the whole stock is to the whole gain or loss, so is each man's stock to his share of the gain or loss.

EXAMPLES.

1. Three merchants trading in company, gained $120: A's stock was $140, B's $300, and C's $160; what was each man's share of the gain? *Ans.* $28 *A's share*, $60 *B's share*, $32 *C's share.*

EXPLANATIONS.

You first add together the different sums which compose the stock in trade, which amounts to $600; then say, as $600, the whole stock in

A's stock $140
B's stock $300
C's stock $160
———
whole stock, $600

trade, upon which the gain was made, is to $120, the whole gain, so is each man's stock to his share of the gain, or $120. It is perfectly plain that each man's share of the gain is, by this operation, in proportion to his stock.

$$\overset{\$}{600} : 120 :: \begin{cases} \overset{\$}{140} : \overset{\$}{28} \text{ A's share.} \\ 300 : 60 \text{ B's share.} \\ 160 : 32 \text{ C's share.} \end{cases}$$

2. Three merchants, in company, have a stock, of which A put in £20, B £30, and C £40, and they gained £360; what was each man's share of the gain? *Ans.* A £80, B £120, C £160.

3. A and B loaded a vessel with 500*hhd.* of sugar; A owned 350*hhd.*, and B 150*hhd.*; in a storm, the captain of the vessel was under the necessity of throwing 100*hhd.* overboard; how many hogsheads must each man lose? *Ans.* A 70*hhd.* and B 30*hhd.*

4. Three merchants F, L, and M, traded together; F put in stock amounting to $500, L $400, M $300, and by misfortune they lost $300; what is each man's share of the loss? *Ans.* F $125, L $100, M $75.

5. A bankrupt is indebted to A $345, to B $255, and to C $400, and his whole property is only $250; what will be each share of the creditor's property? *Ans.* A $86,25*c.* B $63,75*c.* C $100.

COMPOUND FELLOWSHIP.

Q. What is COMPOUND FELLOWSHIP?

A. Compound Fellowship is when the several stocks are continued for different lengths, or unequal terms of time.

RULE.

Q. How do you state and work the terms to find the share of each man's gain or loss?

A. First multiply each man's stock by the time it was continued in trade, and add the several products together; then, as the whole sum of the products is

to the whole gain, or loss, so is each man's product, to his share of the gain or loss.

EXAMPLES.

1. Three merchants, A, B, and C, traded in company; A put in \$600 for 9*mo.*, B \$700 for 12*mo.*, and C \$800 for 15*mo.*, and they gained \$212,10*c.*; what is each man's share of the gain? *Ans.* A \$44,39,3*m.*, B \$69,05,5*m.*, C \$98,65,1*m.*

EXPLANATIONS.

\$	*mo.*	
600 ×	9 =	5400
700 ×	12 =	8400
800 ×	15 =	12000
		25800 amount of products.

\$ *c.*		\$ *c.* *m.*	
25800 : 212,10 : :	5400 :	44, 39, 3	A's share.
	8400 :	69, 05, 5	B's share.
	12000 :	98, 65, 1	C's share
		\$212, 09, 9*m.* proof.	

2. Three merchants traded in company; D put in \$88 for 3*mo.*, E \$120 for 4*mo.*, and F \$300 for 6*mo.*, and they gained \$184; what was each man's share of the gain? *Ans.* D's \$19,09,4*m.*, E's \$34,71,6*m.*, F's \$130,18,8*m.*

3. A and B traded; A put in \$390 for 9*mo.*, and B put in \$215 for 6*mo.*, and by misfortune they lost \$200; what share of the loss must each man sustain? *Ans.* A \$146,25*c.*, B \$53,75*c.*

4. Two merchants traded in company; A put in \$100 for 6*mo.*, and then put in \$50 more; B put in \$200 for 4*mo.* and then took out \$80; at the end of 12*mo.* they had gained \$95; what was each man's share of the gain? *Ans.* A's \$43,71,1*m.*, B's 51,28,8*m.*

ALLIGATION.

Q. What is ALLIGATION?

A. Alligation teaches how to compound or mix together several simples of different qualities, so

that the composition may be of some middle or intermediate quality or price.

EXPLANATIONS.

As was stated on page 168, Alligation is only an application of the Rule of Three Direct. It consists of two kinds; Alligation Medial and Alligation Alternate.

ALLIGATION MEDIAL.

Q. What is ALLIGATION MEDIAL?

A. Alligation Medial teaches, when the quantities and prices of several things are given, to find the mean price or quality of the mixture composed of those materials.

RULE.

Q. How do you state and work the terms to find the mean price or quality of any part of the composition, when the several quantities and their prices or qualities are given?

A. The quantity of each ingredient must first be multiplied by its price or quality; then all the products must be added together, and all the quantities must also be added together into another sum. Then, as the whole composition is to the whole value, or sum of the products, so is any part of the composition to its mean price or quality.

EXAMPLES.

1. A farmer mixed 12*bu.* of rye at 70*c.* a bushel, 15*bu.* of corn at 64*c.* a bushel, and 20*bu.* of barley at 40*c.* a bushel; what is a bushel of this mixture worth? *Ans.* 52*c.* 1*m.*

17*

EXPLANATIONS.

bu.	c.	$ c.
12 at	70	× 8,40
15 at	54	× 8,10
20 at	40	× 8,00
47 bu.		$24,50

bu. $ c. bu.
47 : 24, 50 : : 1
1

c. m.
47)2450(52, 1
235

100
94

60
47

13

In this example, you must first multiply each number of bushels by the price per bushel, and then add the amount together, and also the amount of the bushels contained in the mixture; and then say, as 47 bushels, the whole amount of bushels in the mixture, is to $24 50*c*., the value of the whole composition, or materials, so is 1*bu.* to the mean price or value of one bushel of the composition. The principle of this operation is very plain; for it is perfectly evident, that as the whole number of bushels is to the money it cost, so is 1*bu.* to the value or cost of 1*bu.*

2. A grocer mixed three kinds of tea; 20*lb.* at 5*s.*, 35*lb.* at 8*s.*, and 25*lb.* at 4*s.*; what is a pound of the mixture worth? *Ans.* 6*s.*

3. A silversmith melted 4*oz.* of silver worth 75*c.* an ounce, with 8*oz.* of silver worth 60*c.* an ounce; what is an ounce of this mixture worth? *Ans.* 65*c.*

4. A wine merchant mixed three kinds of wine; 16 *gal.* at $1,10*c.* a gallon, 12*gal.* at 75*c.* a gallon, and 24 *gal.* at 90*c.* a gallon; what is a gallon of this mixture worth? *Ans.* 92*c.* 6*m.*

5. A refiner mixed 2*lb.* of gold of 17 carats fine, with 4*lb.* 23 carats fine; what was the fineness of the mixture? *Ans.* 21 carats.

ALLIGATION ALTERNATE.

Q. What is ALLIGATION ALTERNATE?

A. Alligation Alternate teaches to find what quantity of each of any number of ingredients or simples,

whose rates are given, will compose a mixture of a given rate.

EXPLANATIONS.

The same question, in this rule, often admits of different answers, and it is, therefore, called Alligation *Alternate*. It is the reverse of Alligation Medial.

RULE.

Q. When the prices of the several ingredients or simples are given, how do you find how much of each, at their respective rates, must be taken to make a mixture or compound, at any proposed price?

A. The rates of the ingredients or simples must be placed in a column under each other, with the mean price at the left hand; then each rate which is less than the mean rate must be connected with one or more that is greater, and the difference between each rate and the mean price must be taken and placed directly opposite that rate with which it is connected. If only one difference stand against either rate, it will be the quantity required at that rate; but if there be two or more, their sum will express the quantity.

EXAMPLES.

1. A merchant has oats at 30c. a bushel, barley at 44c. a bushel, corn at 48c. a bushel, and rye at 56c. a bushel; how many bushels of each sort must he mix, that the mixture may be worth 46c. a bushel?

EXPLANATIONS.

In this example, you connect, in the first operation, the 30 and 56, and the 44

	c.	bu.		c.	bu.
mean rate 46c.	30	10 of oats.	or, 46	30	2 oats.
	44	2 barley.		44	10 barl.
	48	2 corn.		48	16 corn.
	56	16 rye.		56	2 rye.

and 48, as 30 and 44 are less than 46, the mean rate; and you set down 10 opposite 30, 16 opposite 56, 2 opposite 44, and 2 opposite 48, as the difference between 46, the mean rate, and the several separate simples. You will readily perceive, that, by this operation, you connect a rate which is less than the mean rate with one that is greater than the mean rate, and set down the difference between them and the mean rate alternately, in such a manner, that there is precisely as much gained by one quantity as there is lost by the other; and that, therefore, the gain and loss on the whole are equal.

2. A grocer has three kinds of sugar; at 26*c.*, 23*c.*, and 20*c.* a pound, how many pounds of each kind must he mix, that the mixture may be worth 22*c.* a pound? *Ans.* 2*lb.* at 26*c.*, 2*lb.* at 23*c.*, and 5*lb.* at 20*c.*

3. A grocer wishes to mix four kinds of tea, at 60*c.*, 70*c.*, 80*c.*, and 90*c.* a pound, in such a manner that the mixture will be worth 75*c.* a pound; how many pounds of each kind must he mix? *Ans.* 15*lb.* at 60*c.*, 5*lb.* at 70*c.*, 5*lb.* at 80*c.*, and 15*lb.* at 90*c.*

RULE.

Q. When one of the ingredients is limited to a certain quantity, how do you find the several quantities of the rest, in proportion to the given quantity?

A. First take the differences between each price and the mean rate, and set them down alternately, as in the preceding rule; then, as the difference standing against that simple whose quantity is given, is to that quantity, so is each of the other differences, severally, to the several quantities required.

EXAMPLES.

1. A merchant wishes to mix 10*gal.* of brandy at 70*c.* a gallon, with one kind at 48*c.* a gallon, another at 36*c.* a gallon, and another at 30*c.* a gallon, so that a gallon of the mixture may be sold for 38*c.*; what quantity of each must be taken?

EXPLANATIONS.

$$\text{Mean rate, } 38\left\{\begin{array}{c} c. \\ 70 \\ 48 \\ 36 \\ 30 \end{array}\right. \begin{array}{l} \\ 8 \text{ stands against the given quantity.} \\ 2 \\ 10 \\ 32 \end{array}$$

$$\text{As } 8 : 10 :: \begin{cases} 2 : 2gal.\ 2qt. \text{ at } 48c. \text{ a gallon.} \\ 10 : 12gal.\ 2qt. \text{ at } 36c. \\ 32 : 40gal.\ 0qt. \text{ at } 30c. \end{cases}$$

2. A grocer wishes to mix 3*lb.* of sugar at 7*c.* a pound, with one kind at 4*c.*, another at 5*c.*, and another at 8*c.* a pound, so that a pound of the mixture may be sold for 6*c.* a pound; how many pounds of each must be taken? *Ans.* 3*lb.* at 7*c.*, 6*lb.* at 4*c.*, 3*lb.* at 5*c.*, and 6*lb.* at 8*c.* a pound.

RULE.

Q. How do you state and work the terms when the whole composition is limited to a given or certain quantity?

A. First find the differences between each price and the mean rate, and set them down, as directed in the preceding rule: then, as the sum of the quantities, or differences thus found, is to the given quantity, or whole composition, so is each ingredient, thus found, to the required quantity of each.

EXAMPLES.

1. A grocer has four kinds of sugar, at 1*c.* 2*c.* 6*c.* and 9*c.* a pound, which he wishes to mix together to make a composition of 96*lb.* at 3*c.* a pound; how many pounds of each must he take?

EXPLANATIONS.

$$\begin{array}{cc} c. & lb. \\ 3\begin{cases} 1 \\ 2 \\ 6 \\ 9 \end{cases} & \begin{array}{r} 6 \\ 3 \\ 1 \\ 2 \\ \hline \end{array} \\ \text{Sum} & 12 \end{array} \qquad \text{As } 12 : 96 :: \begin{array}{c} lb.\ \ lb.\ \ \ \ c. \\ \left.\begin{cases} 6 : 48 \text{ at } 1 \\ 3 : 24 \quad\ \ 2 \\ 1 : \ \ 8 \quad\ \ 6 \\ 2 : 16 \quad\ \ 9 \end{cases}\right\} \text{a } lb. \end{array}$$

2. A grocer wishes to mix water, at 0 a gallon, with brandy, at \$1,60*c.* a gallon, to make a mixture of 120*gal.*, at \$1,20*c.* a gallon; how much of each must he take? *Ans.* 30*gal.* of water, and 90*gal.* of brandy.

DOUBLE RULE OF THREE.

Q. What is DOUBLE RULE OF THREE?

A. The Double Rule of Three teaches to resolve, by one statement, such questions as require two or more statements in single proportion, or Single Rule of Three. In this rule there are always five terms given to find a sixth; the first three terms of which are a supposition, and the last two a demand.

RULE.

Q. How do you state and work the terms in the Double Rule of Three?

A. First reduce the terms, if in different denominations, as in Single Rule of Three. Then, in stating the questions, place the terms of supposition so that the principal cause of loss, gain, or action, possess the first place; that which expresses time, distance of place, the second place; and that which expresses the gain, loss, or action, the third place. Then place the terms of demand under those of the same kind in the supposition. If the blank place or term sought fall under the third term, the proportion is direct; then multiply the first and second terms together for a divisor, and the other three for a dividend, and the quotient will be the answer in the same denomination of the term directly above the blank. But if the blank fall under the first or second term, the proportion is inverse; then multiply the third and fourth terms together for a divisor, and the other three for a dividend, and the quotient will be the answer.

EXAMPLES.

1. If $100, in 12*mo.*, gain $6, what will $200 gain in 8*mo.*? *Ans.* $8.

EXPLANATIONS.

In this example, the blank falls under the third term, and you must, therefore, multiply the first and second terms together for a divisor, and the third, fourth, and fifth, for a dividend, and the quotient is the answer in the same denomination of the third term under which the blank falls.

```
  $    mo.   $
 100 : 12 : : 6  terms of supposition.
 200    8        terms of demand.
       200
       ----
       1600          $
         6          100 first term
      ------         12 second term.
12 | 00)96 | 00     ----
      ------        1200
       $8 Ans.
```

2. If 20 men spend $18 in 24 weeks, how much will 40 men spend in 48 weeks? *Ans.* $72.

3. If the wages of 6 persons, for 21 weeks, be $288, what must 14 persons receive for 46 weeks? *Ans.* $1472.

4. If the carriage of 8*cwt.* 128 miles cost $12,80*c*, what will it cost for the carriage of 12*cwt.* 160 miles? *Ans.* $24.

5. If 10*bu.* of oats be sufficient for 18 horses 20 days, how many bushels will serve 60 horses 36 days? *Ans.* 60*bu.*

6. If 12*bu.* of wheat are sufficient for a family of 4 persons 9*mo.*, how many bushels will be sufficient for a family of 8 persons 12*mo.*? *Ans.* 32*bu.*

7. If 8 men can make 72 rods of wall in 6 days, how many men can make 54 rods in 3 days? *Ans.* 12 *men.*

8. If $700, in 6*mo.*, gain $14 interest, how much will $400 gain in 5 years? *Ans.* $80.

9. What principal, at 7 per cent. per annum, will gain $42 interest in 9*mo.*? *Ans.* $800.

10. If 20 cows, for $80, can be kept 40 weeks, how many cows can be kept 12 weeks for $30? *Ans.* 25 *cows.*

11. If 7 men can reap 84 acres of wheat in 12 days, how many days will it require 20 men to reap 100 acres? *Ans.* 5*da.*

12. If a footman travel 720 miles in 27 days, in travelling 16 hours each day, how many days will he require to travel 240 miles if he travel 12 hours in each day? *Ans.* 12*da.*

EQUATION OF PAYMENTS.

Q. What is EQUATION OF PAYMENTS?

A. Equation of Payments teaches how to find one mean or equated time for the payment of several debts, due at different times, so that no loss shall be sustained by either party.

RULE.

Q. How do you state and work the questions in Equation of Payments?

A. Each payment must be multiplied by its time, and the several products must be added together; then, the sum of the products must be divided by the whole debt, and the quotient will be the answer, or equated time for the payment of the whole.

EXAMPLES.

1. A owes B $1600, of which $800 are to be paid in 4*mo.*, and $800 in 8*mo.*; but they agree that the whole shall be paid at one time; what is the equated time for the payment of the whole? *Ans.* 6*mo.*

EXPLANATIONS.

```
 $      mo.
800  ×  4 = 3200
800  ×  8 = 6400
----        ----
1600)       9600 ( 6mo. Ans.
            9600
            ----
```

In this example, you multiply the 800 by 4, and again by 8, and add the two products for a dividend, and the two payments, making 1600 for a divisor. The principle of this operation is perfectly plain; for, if B shall extend the payment of one half of his debt two months after it is due, he should, unquestionably, receive the other half two months before it is due.

2. C owes D $2000, of which $400 are now due, $800 to be paid in 5*mo.*, and $800 at 15*mo.*, and they agree to make one payment of the whole; at what time must it be paid? *Ans.* 8*mo.*

3. A merchant purchased goods, amounting to $4000, of which $800 are to be paid present, $1600 at 5*mo.*, and the balance, $1600, at 10*mo.*; but they agree to make one payment of the whole; what is the equated time? *Ans.* 6*mo.*

ANNUITIES.

Q. What is an ANNUITY?

A. An Annuity is a sum of money payable every year, for a certain number of years, or for ever.

EXPLANATIONS.

As was stated on page 168, this rule is merely a particular application of the Rule of Three Direct. *Amount* is the sum of the annuities for the time it has been forborne, with the interest due on each payment or annuity. *Present worth* of an annuity is such a sum as being now put to interest, would exactly pay the annuity when it becomes due; and, it is such a sum as must be given for the annuity, if it be paid at the commencement *Contingent* annuity is when the annuity depends on some contingency, as the life or death of a person. *Reversion* is when the annuity does not commence until a number of years has elapsed. *Arrear* is when the debtor keeps the annuity beyond the time of payment.

RULE.

Q. How do you find the amount of an annuity at simple interest?

A. First find the interest of the given annuity for one year; and then for 2, 3, 4, &c., up to the given number of years, less 1: then multiply the annuity by the given number of years, and add the product to the whole interest, and the sum will be the amount sought.

EXAMPLES.

1. What is the amount of an annuity of \$100 for 5 years, simple interest, computed at 7 per cent? *Ans.* \$570.

EXPLANATIONS.

The interest of \$100, at 7 per cent.	for 1 year, is	\$ 7.
	for 2 years,	14.
	for 3 years,	21.
	for 4 years,	28.
Five years' annuity, at \$100 a year is	\$100×5=	500
		Ans. \$570

2. What is the amount of an annuity of \$800, for 5 years, simple interest, computed at 7 per cent? *Ans.* \$4560.

3. A man let a house upon a lease for 8 years, at $200 per annum, and the rent being in arrear for the whole term; what sum must he demand at the end of the term, simple interest being allowed at 6 per cent? *Ans.* $1936.

RULE.

Q. How do you find the present worth of an annuity at Simple Interest?

A. First find the present worth of each year by itself, discounting from the time it becomes due; then the sum of all these will be the answer or present worth required.

EXAMPLES.

1. What is the present worth of $200 per annum, to continue 4 years, at 7 per cent.? *Ans.* $683,89,1*m.*

EXPLANATIONS.

$		$		$		$ c. m.	
107						186,91,5	present worth for 1 year.
114	:	100	: :	200	:	175,43,8	2 years.
121						165,28,8	3 years
128						156,25,0	4 years.

Ans. $683,89,1*m.* present worth required.

2. What is the present worth of $800 per annum to continue 4 years, at 6 per cent.? *Ans.* $2792,12*c.*

3. What is the present worth of $200 per annum, to continue 3 years, at 4 per cent.? *Ans.* $556,[illegible]

INVOLUTION.

Q. What is INVOLUTION?

A. Involution teaches how to find the *powers* of numbers, by multiplying any given number into itself continually a given number of times: and the several products which arise are called powers.

EXPLANATIONS.

The number denoting the height of the power, is called the index or exponent of that power; thus, the number itself is

called the *first* power, or *root*. If the *first* power be multiplied by *itself*, the product is called the *second* power or *square;* and if the square be multiplied by the first power, the product is called the third power, or cube, &c.; thus, 4 is the root or first power of 4; 16 is the 2d power, or square of 4, produced thus, $4\times4=16$; 64 is the 3d power or cube of 4, produced thus, $4\times4\times4=64$, and so on. Thus, you will readily perceive, that to find the square of any given number, you multiply once; and to find the cube you multiply twice.

EXAMPLES.

1. What is the square, or 2d power of 5? *Ans.* 25.

EXPLANATIONS.

In this example, you merely multiply the 5 by itself, and the product is the answer.

$$\begin{array}{r} 5 \\ 5 \\ \hline 25 \text{ Ans.} \end{array}$$

2. What is the square, or 2d power of 7? *Ans.* 49.
3. What is the cube of 3? *Ans.* 27.
4. What is the 5th power of 4? *Ans.* 1024.
5. What is the cube of 9? *Ans.* 729.
6. What is the square of ,5? *Ans.* ,25.
7. What is the cube of ,5? *Ans.* ,125.
8. What is the square of 17,1? *Ans.* 292,41.
9. What is the square of $\frac{2}{3}$? *Ans.* $\frac{4}{9}$.
10. What is the the cube of $\frac{2}{3}$? *Ans.* $\frac{8}{27}$.

NOTE.—A decimal fraction is raised to any power, the same as a whole number, and the same rules are observed in pointing off as in multiplication of decimals. A vulgar fraction is raised to any power by multiplying the numerator of the fraction by itself, and the denominator by itself, until, as in whole numbers, the number of multiplications be one less than the index, or exponent of the power to be found.

TABLE *of the powers of the* 9 *digits, from the* 1*st to the* 5*th.*

Roots,	1	2	3	4	5	6	7	8	9
Squares,	1	4	9	16	25	36	49	64	81
Cubes,	1	8	27	64	125	216	343	512	729
Biquadrates	1	16	81	256	625	1296	2401	4096	6561
Sursolids,	1	32	243	1024	3125	7776	16807	32768	59049

EVOLUTION.

Q. What is EVOLUTION?

A. Evolution is the extracting or finding the root of any given power or number.

EXPLANATIONS.

Evolution is, as you will at once perceive, the reverse of Involution. The root, you have seen, is that number which, being multiplied into itself continually, will produce the given power. The Square Root of any given number is a number which, being multiplied into itself, will produce that number. The Cube Root is a number, which, being cubed, or involved to the third power, will produce the same given number. The power of any given number, or root, may be found exactly by multiplying the number continually into itself; yet, there are numbers, a proposed root of which can never be exactly found; but, by means of decimals, you may approximate or come near to the root, to any degree of exactness. Those numbers whose exact roots can not be obtained are called *surd* numbers; and those whose roots can be exactly found, are called *rational* numbers. This character $\sqrt{}$ placed before any number, expresses the square root of that number; thus, $\sqrt{25}$ expresses the square root of 25. The same character is made to express any other root, by placing the index of the root above it. Thus, $\sqrt[3]{27}$ expresses the cube root of 27; and $\sqrt[4]{625}$ expresses the fourth root of 625, &c. Thus, you can always tell how many figures there will be in the SQUARE ROOT of any number, by pointing it off from unit's place, into periods of two figures each. You can likewise ascertain how many figures there will be in the CUBE ROOT of any number, by pointing it off from unit's place, into periods of three figures each.

EXTRACTION OF THE SQUARE ROOT.

Q. What is extraction of the Square Root?

A. Extraction of the Square Root is to find a number, which, being multiplied into itself, will produce the given number.

RULE.

Q. How do you extract the square root of any given number?

A. 1. Distinguish the given number into periods of two figures each, by putting a point over the place of units, another

over the place of hundreds, and so on over every second figure, both to the left hand in integers, and to the right hand in decimals, which points will show the number of figures the root will consist of.

2. Find the greatest square number in the first period, on the left hand, and set its root on the right hand of the given number, (after the manner of a quotient in division,) for the first figure of the root, and the square number, under the period, and substract it therefrom; and to the remainder bring down the two figures of the next following period for a dividend.

3. Place the double of the root, already found, on the left hand of the dividend for a divisor. Seek how often the divisor is contained in the dividend, (excepting the right hand figure,) and place the figure in the root for the second figure of it, and, likewise, on the right hand of the divisor; multiply the divisor, with the last figure annexed, by the last placed in the root, and substract the product from the dividend; to the remainder join the next period for a new dividend.

4. Double the figures already found in the root for a new divisor, (or bring down your last divisor for a new one, doubling the right hand figure of it,) and from these find the next figure in the root, as last directed, and continue the operation in the same manner, till you have brought down all the periods.

EXAMPLES.

1. What is the square root of 625? *Ans.* 25.

EXPLANATIONS.

```
    625(25
    4
    ---
45)225
   225
   ---
```

In this example, the greatest square in the left hand period, 6, is 4, of which the root is 2, which must be placed in the quotient, and its square, 4, must be substracted from the period, 6, and to the remainder, 2, the next period, 25, must be brought down, making 225. You must then double the root, 2, and place the double, 4, at the left hand of the divisor, and you will find, that 4 is contained in 22, the two left hand figures, 5 times; and you must place it, the 5, both in the root, the quotient, and in the divisor; and then you must proceed as in Simple Division, and you will find the quotient, or root, 25, and no remainder, and then the work is done.

2. What is the square root of 729? *Ans.* 27.

3. What is the square root of 1296? *Ans.* 36.

4. What is the square root of 106929? *Ans.* 327.

5. What is the square root of 10342656? *Ans.* 3216.

6. What is the square root of 6,9169? *Ans.* 2,63.

7. What is the square root of ,001296? *Ans.* ,036.

NOTE.—The root of a vulgar fraction is found by reducing it to its lowest terms, and extracting the root of the numerator for a new numerator, and of the denominator for a new denominator. If the fraction be a surd, reduce it to a decimal and extract its root.

8. What is the square root of $\frac{4}{9}$? *Ans.* $\frac{2}{3}$.

9. What is the square root of $\frac{18}{49}$? *Ans.* $\frac{4}{7}$.

10. What is the square root of $20\frac{1}{4}$? *Ans.* $4\frac{1}{2}$.

11. What is the square root of $\frac{2704}{4225}$? *Ans.* $\frac{4}{5}$.

12. What is the square root of $\frac{42}{70}$? *Ans.* ,7745.

APPLICATION OF THE SQUARE ROOT.

1. A general has an army of 567009 men, how many must he place in rank and file to form them into a square? *Ans.* 753.

2. Bonaparte's army consisted of 490000 men; when brought into a square how many stood in front? *Ans.* 700.

3. If the area of a circle be 1521, what is the side of a square equal in area thereto? *Ans.* 39.

4. A square pavement contains 24336 square stones of equal size; how many are contained in one of its sides? *Ans.* 156.

5. If 1369 fruit-trees be planted in a square orchard, how many must be in a row each way? *Ans.* 37.

6. A square field contains 2025 square rods; how many rods does it measure on each side? *Ans.* 45.

NOTE.—To find a mean proportional between two numbers, you must multiply the given numbers together, and extract the square root of the product.

7. What is the mean proportional between 24 and 96? *Ans.* 48.

8. What is the mean proportional between 49 and 64? *Ans.* 56.

NOTE.—The area of a circle is in proportion to the square of its diameter: multiply the square of the diameter by the given ratio, and the square root of the product will be the answer.

9. If the diameter of a circle be 12 feet, what is the diameter of one $\frac{1}{4}$ as large? *Ans.* 6*ft.*

10. A gentleman has two circular ponds in his pleasure ground; the diameter of the less is 100 feet, and the greater is three times as large; what is its diameter? *Ans.* 173,2.

NOTE.—The square of the hypotenuse, or the longest side of a right angled triangle, is equal to the sum of the squares of the other two sides; and, consequently, the difference of the squares of the hypotenuse and either of the other sides, is the square of the remaining side.

11. What is the length of a ladder that will reach from the top of a wall 32*ft.* high, to the opposite side of a ditch 24*ft.* wide? *Ans.* 40*ft.*

12. The length of the rafters of a certain building is 20*ft.*, and the roof is raised in the centre 12*ft.*; what is the breadth of the building? *Ans.* 32*ft.*

13. If a man travel 40 miles due north, and then turn and travel 30 miles due west, how far will he be from the place from which he first started? *Ans.* 50*m.*

14. What is the distance between two opposite corners of a field 800 rods long, and 600 rods wide? *Ans.* 1000 *rods.*

EXTRACTION OF THE CUBE ROOT.

Q. What is Extraction of the Cube Root?

A. Extraction of the cube root is to find a number, which, being multiplied into its square, will produce the given number.

EXPLANATIONS.

A CUBE is a solid body, having six equal sides, and each of the sides an exact square. The ROOT is, therefore, the measure in length of one of its sides; for, the length, breadth, and thickness of such a cube, body, or square solid, are all alike or equal; and, consequently, the length or root of one side of a cube raised to the 3d power, gives the solid contents.

EXAMPLES.

1. How many solid feet are there in a cubick block, each side measuring 5*ft.*? *Ans.* 125*ft.*

2. How many solid feet are there in a cubick block, each side measuring 8*ft.*? *Ans.* 512*ft.*

RULE.

Q. How do you extract the CUBE ROOT of any given number?

A. 1. Separate the given number into periods of three figures each, by putting a point over the unit figure, and every third figure from the place of units to the left; and, if there be decimals, to the right.

2. Find the greatest cube in the left hand period, and put its root in the quotient.

3 Substract the cube, thus found, from the said period, and to the remainder bring down the next period, and call this the *dividend.*

4. Multiply the square of the quotient by 300, calling it the *divisor.*

5. Seek how many times the divisor may be had in the dividend, and place the result in the quotient or root; then multiply the divisor by this quotient figure, and write the product under the dividend.

6. Multiply the square of this quotient figure by the former *figure* or *figures* of the root, and this product by 30, and place the product under the last; under all, write the cube of this quotient figure, and call the amount the *subtrahend.*

7. Substract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before; and so on, till the whole is finished.

NOTE.—If the divisor can not be had in the dividend, put a cipher in the quotient or root, bring down the next period, and proceed as before directed.

EXAMPLES.

1. What is the cube root of 13824? *Ans.* 24.

EXPLANATIONS.

	13824 (24 root.
$2\times2\times2=$	8
	——
$2\times2\times300=1200$)	5824 dividend.
$1200\times4=$	4800
$4\times4\times2\times30=$	960
$4\times4\times4=$	64
	——
	5824 subtrahend.
	——
	0000

In this example, you first point off the sum into periods of 3 figures each, agreeably to the rule, beginning at the unit's place; then seek the greatest cube in the left hand period, 13, which by the table, you will find is 8, and the root 2; you place the 2, as a quotient, and substract the cube, 8, from the first period, 13; you then bring down the next period, 824, and annex it to the remainder, 5, and call it the dividend: you then multiply the square of the quotient, 2, by 300, for a divisor, which makes 1200; you then seek how many times the divisor is contained in the dividend, which is 4 times; you then multiply the divisor by it, and place the product, 4800, under the dividend; then multiply the square of the last quotient figure, 4, by the quotient obtained before, 2, and that

product by 30, and then place this under the other; and then place the cube of the last quotient figure, 4, under the other two, and add the whole together for a subtrahend, which, substracted from the dividend leaves 0, which shows 24 to be the exact cube of 13824.

2. What is the cube root of 39304? *Ans.* 34.

3. What is the cube root of 941192? *Ans.* 98.

4. What is the cube root of 22069810125? *Ans.* 2805.

5. What is the cube root of ,032768? *Ans.* ,32.

6. What is the cube root of 12,977875? *Ans.* 2,35.

NOTE.—If the root be a surd, reduce it to a decimal before its root is extracted, as in the square root.

7. What is the cube root of $\frac{250}{686}$? *Ans.* $\frac{5}{7}$.

8. What is the cube root of $\frac{8}{27}$? *Ans.* $\frac{2}{3}$.

9. What is the cube root of $\frac{1331}{1728}$? *Ans.* $\frac{11}{12}$.

APPLICATION OF THE CUBE ROOT.

NOTE.—The sides of cubes are as the cube roots of their solid contents; and, therefore, their contents are as the cubes of their sides. The same proportion is true of the similar sides, or of the diameters of all solid figures of similar forms.

EXAMPLES.

1. If a ball, weighing *4lb.*, be *3in.* in diameter, what will be the diameter of a ball, of the same metal, weighing *32lb.*? *Ans. 6in.*

2. The contents of a piece of cubical timber is 103823 solid inches; how many inches is it each way? *Ans. 47in.*

3. There is a cistern of a cubical form, which contains 1331 cubical feet; what are the length, breadth, and depth of it? *Ans.* 11*ft.*

4. How many solid feet of earth must be taken out in digging a cellar, that shall be 12 feet in length, breadth, and depth? *Ans.* 1728*ft.*

5. If the diameter of the sun be 112 times that of the earth, how many globes like the earth would it take to make one as large as the sun? *Ans.* 1404928.

6. The statute bushel contains 2150,4252 cubick or solid inches, what must be the side of a cubick box that shall contain the same quantity? *Ans.* 12,907*in.*

7. If a globe of silver, 4 inches in diameter, be worth $150, what is the value of a globe 8*in.* in diameter? *Ans.* $1200.

A
PRACTICAL SYSTEM
OF
BOOK-KEEPING,
BY SINGLE ENTRY,
FOR FARMERS AND MECHANICKS.

Book-Keeping is the method of recording business transactions. It is of two kinds, single and double entry; but as single entry is the simplest form, it is inserted here as best adapted to the use of farmers and mechanicks. It requires a Day-Book and Leger. Only a few examples are given, merely enough to show the learner the form and manner of keeping books, as it is expected that he will be required to compose similar examples, and insert them in a book for this purpose. Every person, who does not adopt some regular method of keeping his accounts, will be subjected to losses and inconveniences.

DAY-BOOK.

In this book there should be a clear and minute history of business transactions in the order of time in which they occur. At the head of the first page, the owner's name, and the town or city should be placed; after the first page, it will be sufficient to write only the month, day, and year, which should be done in a larger hand than the entries. This book should have head lines, with two columns on the right for dollars and cents, and one on the left for post-marks and references.

LEGER.

In this book all the scattered accounts of the Day-Book should be collected, and all that relates to each individual should be arranged into one separate statement, and placed or posted under his name on two opposite pages of the book, and the accounts entered in which he is debtor on the left hand page, with *Dr.*, and those in which he is credit on the right hand page, with *Cr.* Every Leger should have an alphabetical index, in which the names of the several persons, whose accounts are kept in the Leger, should be written, and the page noted down.

FORM OF A DAY-BOOK.

DAVID WILLIAMS. *New York*, *Jan.* 1, 1832.

			$	c.
1×	*Asa Whitney*, ... *Dr.*			
	For 8*bu.* of wheat *a* 75*c.*		6	00
	5			
1×	*Charles Whiting*, ... *Dr.*			
	For 28*bu.* of corn *a* 50*c.*	$14,00		
	" 15*lb.* of pork *a* 7*c.*	1,05		
	" 8*lb.* of butter *a* 12*c.*	96	16	01
	8			
1×	*David Foster*, ... *Cr.*			
	By 36*gal.* molasses *a* 50*c.*		18	00
	12			
1×	*Asa Whitney*, ... *Dr.*			
	For 37*lbs* of cheese *a* 8*c.*		2	96
	Cr.			
	By 4 days' work *a* 75*c.*		3	00
	18			
1×	*James Atwood*, ... *Cr.*			
	By shoeing horse, ...		2	00
1×	*David Foster*, ... *Dr.*			
	For 50*bu.* of wheat *a* 75*c.*		37	50
	25			
1×	*Asa Whitney*, ... *Cr.*			
	By cash, ...		5	96
1×	*Charles Whiting*, ... *Cr.*			
	By 8 pairs of shoes, *a* $2, ...		16	00
	29			
1×	*James Atwood*, ... *Dr.*			
	For 2*bu.* of wheat *a* $1, ...		2	00

FORM OF A LEGER.

Dr.		Asa Whitney,							Cr.
1832.			$	c.	1832.			$	c.
Jan. 1.	1	For wheat,	6	00	Jan. 12.	1	By work,	3	00
" 12.	1	For cheese,	2	96	" 25.	1	By cash to bal.	5	96
			8	96				8	96

Dr.		Charles Whiting,							Cr.
1832.			$	c.	1832.			$	c.
Jan. 5.	1	For sundries,	16	00	Jan. 25.	1	By shoes	16	00

Dr.		David Foster,							Cr.
1832.			$	c.	1832.			$	c.
Jan. 18.	1	For wheat,	37	50	Jan. 8.	1	By molasses,	18	00
					" 25.		By note to bal.	19	50
								37	50

Dr.		James Atwood,							Cr.
1832.			$	c.	1832.			$	c.
Jan. 29	1	For wheat,	2	00	Jan 18.	1	By work,	2	00

INDEX TO THE LEGER.

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